PROBLEM:

Let $x[n]$ be the complex exponential

$$x[n] = 11e^{j(0.3\pi n + 0.5\pi)}$$

If we define a new signal $y[n]$ to be the output of the difference equation:

$$y[n] = 2x[n] + 4x[n-1] + 2x[n-2]$$

it is possible to express $y[n]$ in the form

$$y[n] = Ae^{j(\omega_0 n + \phi)}$$

Determine the numerical values of A, ϕ and ω_0.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$y[n] = 2x[n] + 4x[n-1] + 2x[n-2]$$

Let $x[n] = 11e^{j(0.3\pi n + 0.5\pi)}$

which also equals $11e^{j0.5\pi}e^{j0.3\pi n}$

Thus,

$$y[n] = 2\left(11e^{j0.5\pi}e^{j0.3\pi n}\right) + 4\left(11e^{j0.5\pi}e^{j0.3\pi(n-1)}\right) + 2\left(11e^{j0.5\pi}e^{j0.3\pi(n-2)}\right)$$

FACTOR OUT $11e^{j0.5\pi}$ and $e^{j0.3\pi n}$

$$y[n] = 11e^{j0.5\pi}e^{j0.3\pi n}\left(2 + 4e^{-j0.3\pi} + 2e^{-j0.6\pi}\right)$$

THIS IS FREQUENCY RESPONSE $\mathcal{H}(\hat{\omega})$ EVALUATED AT $\hat{\omega} = 0.3\pi$

WE NEED PHASOR ADDITION TO GET THE ANSWER

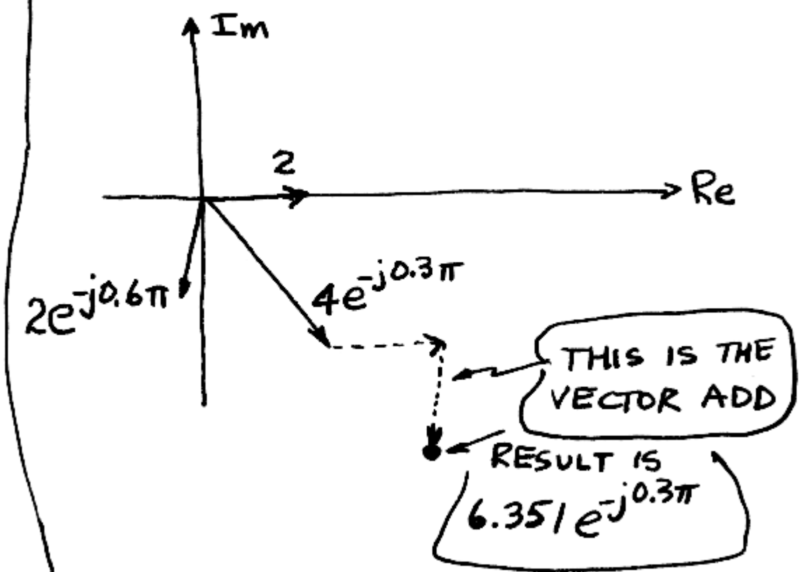
$$y[n] = 11e^{j0.5\pi}e^{j0.3\pi n}\,6.35e^{-j0.3\pi}$$

$$= 69.86e^{j0.2\pi}e^{j0.3\pi n}$$

$A = 69.86$

$\varphi = 0.2\pi$

$\hat{\omega}_0 = 0.3\pi$



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