PROBLEM:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{3} (0.5)^k x[n-k]$$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use the above difference equation to compute the output $y[n]$ when the input is

$$x[n] = \begin{cases} 0 & n < 0 \\ (n+1) & 0 \le n \le 4 \\ -4 & 5 \le n \le 10 \\ 0 & n \ge 11. \end{cases}$$

Make a plot of both $x[n]$ and $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv( )` function.)

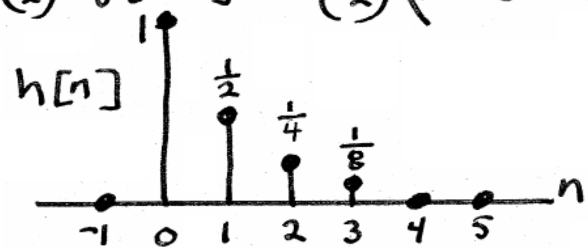
McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

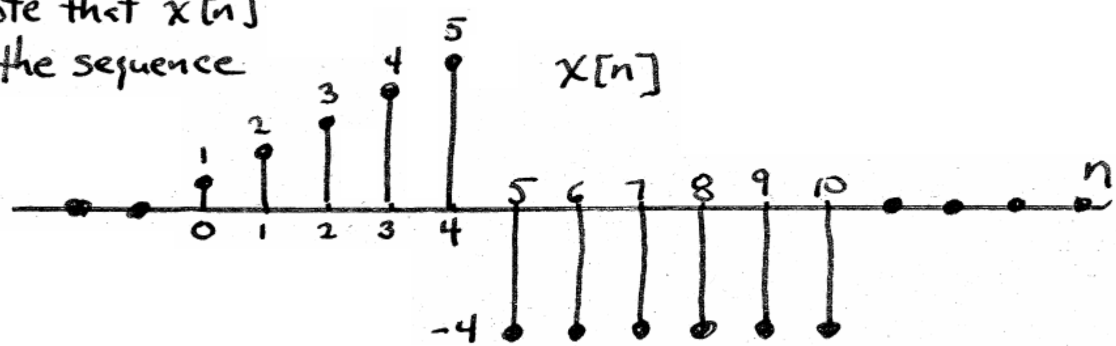
a) $$y[n] = \sum_{k=0}^{3} \left(\tfrac{1}{2}\right)^k x[n-k] = \sum_{k=0}^{3} b_k\, x[n-k]$$

$$b_k = \begin{cases} \left(\tfrac{1}{2}\right)^k & 0 \le k \le 3 \\ 0 & \text{otherwise} \end{cases}$$

b) $$h[n] = \sum_{k=0}^{3} \left(\tfrac{1}{2}\right)^k \delta[n-k] = \left(\tfrac{1}{2}\right)^n \left(u[n] - u[n-4]\right)$$



c) Note that x[n] is the sequence



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The difference equation can be evaluated easily by using a chart. $y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] + \frac{1}{8}x[n-3]$

n	$x[n]$	$\frac{1}{2}x[n-1]$	$\frac{1}{4}x[n-2]$	$\frac{1}{8}x[n-3]$	$y[n]$
0	1	0	0	0	1
1	2	$\frac{1}{2}$	0	0	$2\frac{1}{2}$
2	3	1	$1/4$	0	$4\frac{1}{4}$
3	4	$3/2$	$1/2$	$1/8$	$6\frac{1}{8}$
4	5	2	$3/4$	$1/4$	8
5	-4	$5/2$	1	$3/8$	$-\frac{1}{8}$
6	-4	-2	$5/4$	$1/2$	$-4\frac{1}{4}$
7	-4	-2	-1	$5/8$	$-6\frac{3}{8}$
8	-4	-2	-1	$-\frac{1}{2}$	$-7\frac{1}{2}$
9	-4	-2	-1	$-\frac{1}{2}$	$-7\frac{1}{2}$
10	-4	-2	-1	$-\frac{1}{2}$	$-7\frac{1}{2}$
11	0	-2	-1	$-\frac{1}{2}$	$-3\frac{1}{2}$
12	0	0	-1	$-\frac{1}{2}$	$-1\frac{1}{2}$
13	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
14	0	0	0	0	