PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

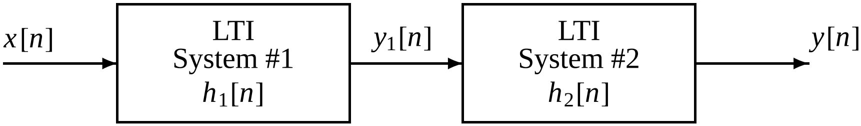


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a seven-point running average filter,

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 1/7 & n = 0, 1, 2, 3, 4, 5, 6 \\ 0 & n > 6 \end{cases}$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] - y_1[n-1]$$

We can find the impulse response of the overall system, by first finding the impulse response of the first system and then use that response as the input to the second system.

(a) Determine the filter coefficients of System #1, and also for System #2.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $y_1[n]$ and make a plot.

(c) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$. Take advantage of the fact that you already know the output of System #1, $y_1[n]$, when its input is an impulse.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) For System #1,

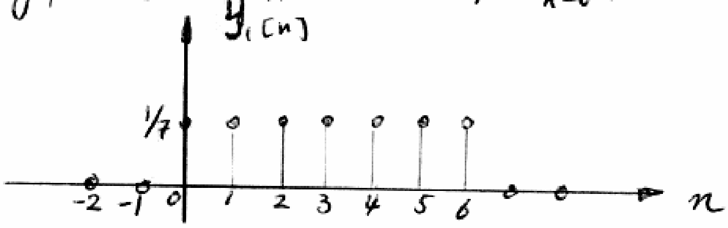
$$b_n = \begin{cases} 0 & n < 0 \\ 1/7 & n = 0,1,2,3,4,5,6 \\ 0 & n > 0 \end{cases}$$

For System #2,

$$b_n = \begin{cases} 1 & n = 0 \\ -1 & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$y_1[n] = h_1[n] = 1/7 \sum_{k=0}^{6} \delta[n-k]$$



(c)

$$\begin{aligned} y[n] &= y_1[n] - y_1[n-1] \\ &= 1/7 \sum_{k=0}^{6} \delta[n-k] - 1/7 \sum_{k=0}^{6} \delta[n-1-k] \\ &= 1/7 \sum_{k=0}^{6} \delta[n-k] - 1/7 \sum_{k=1}^{7} \delta[n-k] \\ &= 1/7\, \delta[n] - 1/7\, \delta[n-7] \end{aligned}$$

Since $y[n]$ is the output of the cascade system when input is a impulse $\delta[n]$, the impulse response of the system is

$$h[n] = y[n] = \begin{cases} 1/7 & n = 0 \\ -1/7 & n = 7 \\ 0 & \text{otherwise} \end{cases}$$