

Early Learning – Primary Content Standards



for

Mathematics



State Board of Education of Ohio

Jennifer L. Sheets, Pomeroy, *President*

Jennifer H. Stewart, Zanesville, *Vice President*

John R. Bender, Avon
Virgil E. Brown, Jr., Shaker Heights
Deborah Cain, Uniontown
Michael Cochran, Blacklick
Colleen D. Grady, Strongsville
Lou Ann Harrold, Ada
Susan Haverkos, West Chester
Heather Heslop Licata, Akron
Robin C. Hovis, Millersburg
Stephen M. Millett, Columbus
Eric Okerson, Cincinnati
Emerson J. Ross, Jr., Toledo
G. R. "Sam" Schloemer, Cincinnati
Jane Sonenshein, Loveland
Sue Westendorf, Bowling Green
Carl Wick, Centerville
Ann Womer Benjamin, Aurora

Ex Officio Members

The Honorable Joy Padgett, Coshocton
The Honorable Arlene J. Setzer, Vandalia

Ohio Department of Education

Susan Tave Zelman,
Superintendent of Public Instruction

Center for Students, Families and Communities

Jane Wiechel, *Associate Superintendent*

Office of Early Learning and School Readiness

Sandra Miller, *Director*
Jeanne Lance, *Assistant Director*
Kim Carlson, *Assistant Director*
James Scott, *Assistant Director*
Stephanie Siddens, *Assistant Director*
Debbie McGlothlin, *Project Coordinator*



This document is an official publication of the State Board of Education and the Ohio Department of Education. Information within represents official policy of the State Board.

The Ohio Department of Education does not discriminate on the basis of race, color, national origin, sex, religion, age, or disability in employment or the provision of services.



Mathematics

Foreword

In response to Amended Substitute House Bill 94 of 2001, the Ohio Department of Education's Office of Early Learning and School Readiness and the Office of Curriculum and Instruction convened Ohio stakeholders to develop early learning content standards for mathematics.

Membership for the mathematics writing team was selected from a pool of more than 100 nominees representing Ohio early childhood stakeholders. They included Head Start, public and private preschool teachers; kindergarten teachers; program administrators; and faculty members from higher education teacher preparation institutions. The team was balanced to include representation from all geographic areas of the state, as well as ethnic diversity.

The early learning content standards describe essential concepts and skills that young children should know and be able to use at the end of their preschool experience. Based on research, these achievable prekindergarten indicators are relevant to all early learning experiences, regardless of setting (e.g., nursery school, preschool and family care) and are aligned to the kindergarten through 12th-grade indicators, benchmarks and standards that result in a seamless continuum of learning for children preschool through kindergarten and primary grades.

A draft of the early learning content standards for mathematics was disseminated for review and focused feedback from experts within and outside of Ohio. It also was posted on the Ohio Department of Education Web site for broad public input. Based on review of feedback, final revisions to the early learning content standards for mathematics document were completed. The final document was adopted by the State Board of Education in December 2003.



When children's intuitions are respected and valued, and when they are encouraged to listen to other children explain how they answer questions, they naturally pick up more advanced ways of solving problems.

– Author Unknown

Mathematics

Table of Contents



Overview	1
Introduction	5
Standards and Prekindergarten Indicators	9
Indicators by Standard	17
Standards, Benchmarks and Indicators	23
Instructional Commentary	61
Glossary	71
Resources	75



We have seen that children learn the real basics of thinking about mathematics through personal experience and playful activities. With appropriate learning experiences from birth through early elementary years, children will develop a lifelong interest in using mathematics.

– Janet Brown McCracken, 1987

Mathematics

Overview





Mathematics Standards



The mathematics academic content standards provide a set of clear, rigorous expectations for all students and provide teachers with clearly defined statements of what students should know and be able to do as they progress through school. The standards represent a research-based approach and prepare all students for success. Competency in mathematics includes understanding of mathematical concepts, facility with mathematical skills, and application of concepts and skills to problem-solving. Students are able to communicate mathematical reasoning using mathematical and everyday language.

Whenever possible, students should have opportunities to learn mathematics through real-world contexts, including practical applications, real data and numbers often associated with situations and problems encountered in workplace and daily life. All students should be exposed to a mathematics program rich in technology, including calculators, computers and technology applications.

The six standards that follow represent the mathematics content:

Content Standards: Number, Number Sense and Operations
 Measurement
 Geometry and Spatial Sense
 Patterns, Functions and Algebra
 Data Analysis and Probability

Process Standard: Mathematical Processes

The rigorous, yet realistic standards provide a comprehensive foundation for all students to think and reason mathematically and to use mathematics knowledge and skills effectively in post-secondary education, the workplace and daily life. These standards represent a connected body of mathematical understanding and competencies rather than a menu of distinct topics from which to choose.

Throughout the content standards, students will use mathematical processes including reasoning, communication and representation skills and appropriate technology within problem-solving situations. Making connections within mathematics constructs and between mathematics and other disciplines is critical for student success in using mathematics effectively in school, work and daily life.

Though each of these six standards applies to all grades, emphasis will vary both within and between grade clusters. For example, the emphasis on number, number sense and operations is greatest in elementary grades, and by grades nine through 12, topics related to number, number sense and operations represent a smaller portion of the curriculum.

The six standards do not separate the curriculum into distinct topics, since the content described by the standards and benchmarks are interrelated. For example, number pervades all areas of mathematics, and some topics in measurement are closely related to those within geometry. Mathematical processes are used in all content areas and are best learned when carefully integrated with content in instruction and assessment. In addition, the indicators are not listed in any specific order of importance or development.

To describe the general framework of Ohio's standards, the following terms and definitions are used in the document.

Standard: The standard statement describes, in broad terms, what children or students should know and be able to do as a result of the preschool or kindergarten through 12th-grade programs. A standard is an overarching goal or theme.

Benchmark: Benchmarks are specific statements of what all students should know and be able to do at a specified time in their schooling. Benchmarks are used to measure a student's progress toward meeting standards. Mathematics benchmarks are defined for grade bands PreK-2, 3-4, 5-7, 8-10 and 11-12.

Grade-level Indicators: Indicators serve as checkpoints to monitor progress toward the benchmarks.



If children are excited, curious, resourceful and confident about their ability to figure things out and eager to exchange opinions with other adults and children, they are bound to go on learning, particularly when they are out of the classroom and throughout the rest of their lives.



Mathematics Writing Team



The Ohio Department of Education's Office of Early Learning and School Readiness and Office of Curriculum and Instruction express gratitude to the writing team members who contributed their expertise and time developing Ohio's early learning content standards for mathematics. The members devoted many hours to research and to thoughtful consideration of issues to ensure that the standards reflect wise and responsible decisions regarding mathematics content. The writing team members represent the many caring and concerned individuals across the state dedicated to their profession and to high quality early education for all of Ohio's children.

Ruth Al-Esail
Athens City Schools
Kindergarten Teacher

Marcia Barnhart
Leipsic Local Schools
Kindergarten Teacher

Francie Berg
Coshocton City Schools
Preschool Teacher

Margaret Bramel
South-Western City Schools
Administrator

Ann Deal
Sinclair Community College
Preschool Teacher

Brendan Foreman
John Carroll University
Assistant Professor

Eugene Geist
Ohio University
Professor

Jamie Harmount
Highland County Community
Action Organization
Administrator

Patricia Harper
Kent City Schools
Preschool Teacher

Deborah McGlothlin
Ohio Department of Education
Office of Early Learning and
School Readiness
Consultant

Barbara Phillips
Ohio Department of Education
Office of Curriculum and Instruction
Consultant

Lois Popejoy
Toronto City Schools
Preschool Teacher

Fran Rodstrom
Knox County Educational
Service Center
Preschool Teacher

Niketa Starr
Cleveland Municipal Schools
Preschool Teacher

Wendy Tyger
Mahoning County Educational
Service Center
Administrator

Mathematics

Introduction





Introduction



Ohio's mathematics content standards serve as a basis for what all students should know and be able to do by the time they have graduated from high school. These standards, benchmarks and grade-level indicators are intended to provide Ohio educators with a set of common expectations from which to base mathematics curricula.

Ohio's Mathematics Content Standards are intended to:

- Prepare students to make everyday decisions, such as choosing which product to purchase, interpreting information in news reports and selecting insurance or health plans;
- Develop mathematical thinking and problem-solving needed in the workplace. Those who understand and can use mathematics have significantly enhanced opportunities and options; and
- Play a central role in modern culture, including aesthetic and recreational aspects, and an essential role in the scientific and technical community.

Ohio's Early Learning Content Standards are based upon principles that:

- Promote high expectations for mathematical thinking and problem-solving for *all* students;
- Represent early mathematics concepts and skills needed to provide a foundation for successful mathematics instruction in the primary grades;
- Recognize the importance of children representing and demonstrating knowledge and understanding in multiple ways;
- Recognize the influence of diverse cultural environments in the mathematics acquisition of young children;
- Represent essential concepts and skills that can be addressed within the context of meaningful experiences;
- May serve as a framework for planning and implementing early mathematical experiences within the context of daily routines, activities and play; and
- Serve as the basis for classroom, program and state assessments.



Mathematics for All



The Ohio Department of Education believes that Ohio's academic content standards are for all children and students. Clearly defined standards delineate what all college- and career-bound students should know and be able to do as they progress through the grade levels. Well-defined standards ensure that parents, teachers and administrators will be able to monitor students' development. Students, as stakeholders in their own learning, will be capable of tracking their own learning.

No individual or group should be excluded from the opportunity to learn, and all children are presumed capable of learning. Every Ohio student, regardless of race, gender, ethnicity, socioeconomic status, limited English language proficiency, learning disability or giftedness, should have access to a challenging, standards-based curriculum.

The knowledge and skills defined in Ohio academic content standards are within the reach of all students. However, students develop at different rates. Given time and opportunity, all children learn and experience success, but the degree to which the standards are met and the time it takes to reach the standards will vary from student to student.

Students with disabilities should have Individualized Education Programs (IEPs) aligned with the standards. Children with disabilities are first and foremost students of the general curriculum, yet they may require specific supports and interventions, in varying degrees, to progress in the curriculum. Accommodations and modifications provided to children with disabilities are not intended to compromise the content standards. Rather, these supports provide students the opportunity to maximize their strengths, compensate for their learning difficulties, and participate and progress in the standards-based curriculum.

Students who can exceed the grade-level indicators and benchmarks set forth in the standards must be afforded the opportunity and be encouraged to do so. Gifted and talented students may require special services or activities in order to fully develop their intellectual, creative, artistic and leadership capabilities or to excel in a specific content area.

Students with limited English proficiency (LEP) also may need specific supports and adaptive instructional delivery to achieve Ohio's academic content standards. An instructional delivery plan for a student with LEP needs to take into account the student's level of English language proficiency as well as his or her cultural experiences.

Identifying and nurturing the talents of all students and strategizing with students to address educational needs will enable all children to reach the standards. The Ohio Department of Education encourages the early childhood community to align its educational programs with the standards to ensure that all of Ohio's children reach their full potential.



There is no one best way to educate all children. We must discover a child's areas of strengths and characteristic approaches to learning. We must, as much as possible, bring the teaching to where the child is.

– Howard Gardner, 1983

Mathematics

Standards and Prekindergarten Indicators





Ohio's Preschool through Grade 12 Mathematics Standards



The following standards provide guidance to early childhood educators across preschool and child care settings. Note how the standards for early childhood connect with those for grades kindergarten through 12.

Number, Number Sense and Operations

For Kindergarten through Grade 12

Students demonstrate number sense, including an understanding of number systems and operations and how they relate to one another. Students compute fluently and make reasonable estimates using paper and pencil, technology-supported and mental methods.

For Early Childhood

At the heart of mathematics is an understanding of number relationships. Children need to make sense of the ways numbers are used in their everyday world. Number senses and concepts develop gradually over time as young children explore, manipulate and organize materials and as they communicate their mathematical thinking. Counting is one of the earliest number concepts; it begins with developing oral counting skills or rote counting. One-to-one correspondence follows rote counting, which means linking one number, and only one, with each item in a set of objects. Other number concepts addressed within the early mathematics curriculum include quantity, comparisons and number symbols. Quantity is the concept of an entire set: knowing that the last object counted represents the entire set of objects.

Children will begin finding ways to represent numbers. They may make marks or write numerals. Through children's curiosity and involvement in real-life experiences, they come to understand the meaning of number operations and begin making comparisons using terms such as *more than*, *bigger than*, *less than* and *the same as*.

Measurement

For Kindergarten through Grade 12

Students estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units, tools and technologies.

For Early Childhood

Preschool measurement activities focus on developing and understanding the principles and uses of measuring. Children learn measurement from opportunities to use a variety of nonstandard and standard materials for measurement through hands-on activities. As a first step, children often make comparisons without any measurement tools. Using materials provided in their play, children begin to notice materials that are longer, shorter, heavier and lighter. Next, children often demonstrate an interest in measurement through nonstandard tools by using their hand, a piece of string or a ribbon to measure objects and spaces in their world. It

is often through a variety of experiences that children will find a need for more conventional measurement tools. Formal instruction on the uses of standard measures such as clocks, rulers and scales can be introduced in the preschool grades and made available through play.

Geometry and Spatial Sense

For Kindergarten through Grade 12

Students identify, classify, compare and analyze characteristics, properties and relationships of one-, two- and three-dimensional geometric figures and objects. Students use spatial reasoning, properties of geometric objects and transformations to analyze mathematical situations and solve problems.

For Early Childhood

Geometry and spatial sense refer to the recognition of shapes and structures found in the environment. Children learn about and use their knowledge of two- and three-dimensional shapes when given the opportunity to create designs with pattern blocks; draw, paint and cut shapes for their artwork; organize blocks by sorting them; and locate shapes in outdoor settings.

Geometry also involves an understanding of space. Children gain spatial sense as they investigate, experiment and explore everyday objects and physical materials and as they become aware of themselves in relation to the world around them. Children need to feel themselves in space by climbing high or swinging low, and by crawling in, out, on top of and under other objects. Through these experiences, early childhood educators introduce children to the vocabulary of space, question them about their position in space, and help them learn about location and position (on, off, on top of, under, in, out, behind, below), movement (backward, forward, around, through, across, up, down), and distance (near, far, next to).

Patterns, Functions and Algebra

For Kindergarten through Grade 12

Students use patterns, relations and functions to model, represent and analyze problem situations that involve variable quantities. Students analyze, model and solve problems using various representations such as tables, graphs and equations.

For Early Childhood

A pattern is an organized arrangement of shapes and objects. Pattern recognition facilitates children's understanding of the relationships among objects and their ability to make generalizations about number combinations and counting. As a component of algebra, creating and using patterns can be interesting and accessible to young children. Children can begin noticing patterns in the routine of the day, or patterns of colors, shapes or sizes through teacher guidance and comment. Recognizing patterns and relationships is not just an important objective in mathematics, but one that children will use in other content areas, such as science and literacy. For preschoolers, the goal is to recognize and analyze simple patterns, copy them, create them and make predictions about them by extending them.

Data Analysis and Probability

For Kindergarten through Grade 12

Students pose questions and collect, organize, represent, interpret and analyze data to answer those questions. Students develop and evaluate inferences, predictions and arguments that are based on data.

For Early Childhood

Active children learn through active experiences. Data collection, organization, representation and analysis in preschool involve sorting, classifying, graphing, counting, measuring and comparing. Instruction in each of these areas can build on young learners' natural interest in making collections. As part of collecting, children first sort and make sets without any plan in mind. Later, they sort more purposefully, according to properties such as color, shape or size. As children develop and refine their sorting skills, they will sort by more than one attribute. Early childhood educators can strengthen this ability when young children are encouraged to talk about their sorting rules.

Graphing is an extension of sorting and classifying. A graph presents information in a visually organized way that helps children see relationships. While graphing is an abstract concept for young children, simple graphs using concrete (real) objects and later symbolic (pictorial) representations can provide an appropriate and meaningful way to display findings and information. For example, a simple graph of the kinds of shoes children are wearing could develop from a concrete representation (shoes with ties, hook and loop tape, buckles and slip-on shoes), to a symbolic one (pictures representing the types of shoes), or to marks for representing the number of shoes.

Mathematical Processes

For Kindergarten through Grade 12

Students use mathematical processes and knowledge to solve problems. Students apply problem-solving and decision-making techniques and communicate mathematical ideas.

For Early Childhood

Young children are problem solvers. As they explore and examine their world — pulling, pushing, tasting and taking things apart — they are attempting to find out how things work. This natural drive to solve problems should be built upon in the mathematics curriculum. For example, children will need to count the number of cups of sugar to make cookies, the number of children they will need to play a game, or the number of children who can work together in the block area at one time. Within these learning opportunities, real problems are posed, and children are guided to use the mathematical processes of reasoning, communicating, representing and connecting to solve them.

Within the six standards for mathematics, mathematical processes are specified through benchmark statements only. Mathematical process skills are addressed and embedded within the prekindergarten indicators. Young children should be guided to use these processes in problem-solving situations.

Note: *There are no indicators for this standard. Mathematical processes are used in all content areas and should be incorporated within other curriculum content and as part of instructional strategies and assessment procedures, as relevant.*



Mathematics Prekindergarten Indicators



Number, Number Sense and Operations

Number and Number Systems

1. Count to 10 in the context of daily activities and play (e.g., number songs).
2. Touch objects and say the number names when counting in the context of daily activities and play (e.g., cookies on a plate, steps on a set of stairs).
3. Demonstrate one-to-one correspondence when counting objects (e.g., give one cookie to each child in group).
4. Determine “how many” in sets of 5 or fewer objects.
5. Construct two sets of objects, each containing the same number of objects (e.g., 5 crayons and 5 blocks).
6. Compare sets of equal, more and fewer and use the language of comparison (e.g., equal, more and fewer).
7. Group and regroup a given set in the context of daily activities and play (e.g., 5 blocks can be 2 blue and 3 green or 1 blue and 4 green).
8. Represent quantity using invented forms (e.g., child’s marks to represent a quantity of objects).
9. Write numerical representations (e.g., scribbles, reversals) or numerals in meaningful context (e.g., play situations).
10. Identify and name numerals 0-9.
11. Compare and order whole numbers up to 5.
12. Identify penny, nickel, dime and quarter and recognize that coins have different values.

Meaning of Operations

13. Construct sets with more or fewer objects than a given set.
14. Count on (forward) using objects such as cards, number cubes or dominoes that have familiar dot patterns (e.g., when selecting 5 apples from a bag, takes out two and continues counting 3, 4, 5).

Meaning of Operations (continued)

15. Join two sets of objects to make one large set in the context of daily routines and play (e.g., combining 2 bags of raisins, each containing 3 pieces; combining 2 groups of blocks, each containing 3 blocks).
16. Distribute equally a set of objects into 2 or more smaller sets (e.g., shares 6 crackers with 3 friends equally).

Measurement

Measurement Units

1. Begin to identify and use the language of units of time. For example:
 - a. Day, night, week;
 - b. Yesterday, today, tomorrow.

Use Measurement Techniques and Tools

2. Recognize that various devices measure time (e.g., clock, timer, calendar).
3. Sequence or order events in the context of daily activities and play (e.g., wash your hands before and after snacks, who's next for the computer).
4. Begin to use terms to compare the attributes of objects (e.g., bigger, smaller, lighter, heavier, taller, shorter, more and less).
5. Order a set of objects according to size, weight or length (e.g., cups of different sizes).
6. Measure length and volume (capacity) using non-standard units of measure (e.g., how many paper clips long is a pencil, how many small containers it takes to fill one big container using sand, rice or beans).

Geometry and Spatial Sense

Characteristics and Properties

1. Match identical two-and three-dimensional objects found in the environment in play situations (e.g., 2 squares of same size, 2 stop signs).
2. Sort and classify similar two-and three-dimensional objects in the environment and play situations (e.g., paper shapes, 2 balls of different size).
3. Identify, name, create and describe common two-dimensional shapes in the environment and play situations (e.g., circles, triangles, rectangles and squares).
4. Identify, name and describe three-dimensional objects using the child's own vocabulary (e.g., sphere-"ball," cube-"box," cylinder -"can" or "tube," and cone-"ice cream cone").

Spatial Relationships

5. Demonstrate and begin to use the language of the relative position of objects in the environment and play situations (e.g., up, down, over, under, top, bottom, inside, outside, in front, behind, between, next to, right side up and upside down).

Patterns, Functions and Algebra

Use Patterns, Relations and Functions

1. Sort, order and classify objects by one attribute (e.g., size, color, shape, use).
2. Identify, copy, extend and create simple patterns or sequences of sound, shapes and motions in the context of daily activities and play (e.g., creates red, blue, red, blue pattern with blocks).

Use Algebraic Representations

3. Use play, physical materials or drawings to model a simple problem (e.g., There are 6 cookies to be shared by 3 children. How many cookies can each child receive?).

Data Analysis and Probability

Data Collection

1. Gather, sort and compare objects by similarities and differences in the context of daily activities and play (e.g., leaves, nuts, socks).
2. Place information or objects in a floor or table graph according to one attribute (e.g., size, color, shape or quantity).

Statistical Methods

3. Select the category or categories that have the most or fewest objects in a floor or table graph (e.g., favorite ice cream).



Most children are motivated to learn by an intense desire to make sense out of their world and achieve the competencies desired by the culture.

– Sue Bredekamp and Copple (Ed.), 1997

Mathematics

Indicators by Standard

Prekindergarten – Kindergarten





Mathematics Prekindergarten and Kindergarten Indicators

The following lists the prekindergarten and kindergarten indicators within each standard.

I. Number, Number Sense and Operations Standard

Prekindergarten

- Count to 10 in the context of daily activities and play (e.g., number songs).
- Touch objects and say the number names when counting in the context of daily activities and play (e.g., cookies on a plate, steps on a set of stairs).
- Demonstrate one-to-one correspondence when counting objects (e.g., give one cookie to each child in group).
- Determine “how many” in sets of 5 or fewer objects.
- Construct two sets of objects, each containing the same number of objects (e.g., 5 crayons and 5 blocks).
- Compare sets of equal, more and fewer and use the language of comparison (e.g., equal, more and fewer).
- Group and regroup a given set of objects in the context of daily activities and play (e.g., 5 blocks can be 2 blue and 3 green or 1 blue and 4 green).
- Represent quantity using invented forms (e.g., child’s marks to represent a quantity of objects).
- Write numerical representations (e.g., scribbles, reversals) or numerals in meaningful context (e.g., play situations).
- Identify and name numerals 0-9.

Kindergarten

- Count to twenty; e.g., in play situations or while reading number books.
- Explain rules of counting, such as each object should be counted once and that order does not change the number.
- Determine “how many” in sets (groups) of 10 or fewer objects.
- Construct multiple sets of objects each containing the same number of objects.
- Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects.
- Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green.
- Relate, read and write numerals for single-digit numbers (0-9).



I. Number, Number Sense and Operations Standard (continued)

Prekindergarten

- Compare and order whole numbers up to 5.
- Identify penny, nickel, dime and quarter and recognize that coins have different values.
- Construct sets with more or fewer objects than a given set.
- Count on (forward) using objects such as cards, number cubes or dominoes that have familiar dot patterns (e.g., when selecting 5 apples from a bag, takes out two and continues counting 3, 4, 5).
- Join two sets of objects to make one large set in the context of daily activities and play (e.g., combining 2 bags of raisins, each containing 3 pieces; combining 2 groups of blocks, each containing 3 blocks).
- Distribute equally a set of objects into 2 or more smaller sets (e.g., shares 6 crackers with 3 friends equally).

Kindergarten

- Compare and order whole numbers up to 10.
- Identify and state the value of a penny, nickel and dime.
- Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
 - a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two or another small amount.
 - b. Count on (forward) and count back (backward) on a number line between 0-10.
- Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces.
- Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children.

II. Measurement Standard

Prekindergarten

- Begin to identify and use the language of units of time. For example:
 - a. Day, night, week;
 - b. Yesterday, today, tomorrow.
- Recognize that various devices measure time (e.g., clock, timer, calendar).

Kindergarten

- Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days.
- Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less.

II. Measurement Standard (continued)

Prekindergarten

- Sequence or order events in the context of daily activities and play (e.g., wash your hands before and after snacks, who's next for the computer).
- Begin to use terms to compare the attributes of objects (e.g., bigger, smaller, lighter, heavier, taller, shorter, more and less).
- Order a set of objects according to size, weight or length (e.g., cups of different sizes).
- Measure length and volume (capacity) using non-standard units of measure (e.g., how many paper clips long is a pencil, how many small containers it takes to fill one big container using sand, rice, or beans).

Kindergarten

- Order events based on time. For example:
 - a. Activities that take a long or short time;
 - b. Review what we do first, next, last;
 - c. Recall what we did or plan to do yesterday, today, tomorrow.
- Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less.
- Measure length and volume (capacity) using uniform objects in the environment. For example, find:
 - a. How many paper clips long is a pencil;
 - b. How many small containers it takes to fill one big container using sand, rice, beans.



III. Geometry and Spatial Sense Standard

Prekindergarten

- Match identical two- and three-dimensional objects found in the environment in play situations (e.g., 2 squares of same size, 2 stop signs).
- Sort and classify similar two- and three-dimensional objects in the environment and play situations (e.g., paper shapes, 2 balls of different size).
- Identify, name, create and describe common two-dimensional shapes in the environment and play situations (e.g., circles, triangles, rectangles and squares).
- Identify, name and describe three-dimensional objects using the child's own vocabulary (e.g., sphere-“ball,” cube-“box,” cylinder-“can” or “tube,” and cone-“ice cream cone”).
- Demonstrate and begin to use the language of the relative position of objects in the environment and play situations (e.g., up, down, over, under, top, bottom, inside, outside, in front, behind, between, beside, next to, right side up and upside down).

Kindergarten

- Identify and sort two-dimensional shapes and three-dimensional objects. For example:
 - a. Identify and describe two-dimensional figures and three-dimensional objects from the environment using the child's own vocabulary.
 - b. Sort shapes and objects into groups based on student-defined categories.
 - c. Select all shapes or objects of one type from a group.
 - d. Build two-dimensional figures using shapes or tangrams; build simple three-dimensional objects using blocks.
- Name and demonstrate the relative position of objects as follows:
 - a. Place objects over, under, inside, outside, on, beside, between, above, below, on top of, upside-down, behind, in back of, in front of;
 - b. Describe placement of objects with terms, such as on, inside, outside, above, below, over, under, beside, between, in front of, behind.

Playful approaches to higher order cognitive skills such as critical thinking...are especially important during the elementary years as children begin to be able to 'play with ideas,' testing their thinking and comparing it with that of their peers. Opportunities for creating playful challenges that have a comfortable level of risk are crucial during this age period and should be available at school.

– Doris Bergen and Sherri Oden, 1988

IV. Patterns, Functions and Algebra Standard

Prekindergarten

- Sort, order and classify objects by one attribute (e.g., size, color, shape, use).
- Identify, copy, extend and create simple patterns or sequences of sounds, shapes and motions in the context of daily activities and play (e.g., creates red, blue, red, blue pattern with blocks).
- Use play, physical materials or drawings to model a simple problem (e.g., There are 6 cookies to be shared by 3 children. How many cookies can each child receive?)

Kindergarten

- Sort, classify and order objects by size, number and other properties. For example:
 - a. Identify how objects are alike and different.
 - b. Order three events or objects according to a given attribute, such as time or size.
 - c. Recognize and explain how objects can be classified in more than one way.
 - d. Identify what attribute was used to sort groups of objects that have already been sorted.
- Identify, create, extend and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10.
- Model a problem situation using physical materials.

V. Data Analysis and Probability Standard

Prekindergarten

- Gather, sort and compare objects by similarities and differences in the context of daily activities and play (e.g., leaves, nuts, socks).
- Place information or objects in a floor or table graph according to one attribute (e.g., size, color, shape or quantity).
- Select the category or categories that have the most or fewest objects in a floor or table graph (e.g., favorite ice cream).

Kindergarten

- Gather and sort data in response to questions posed by teacher and student; e.g., how many sisters and brothers, what color shoes.
- Arrange objects in a floor or table graph according to attributes, such as use, size, color or shape.
- Select the category or categories that have the most or fewest objects in a floor or table graph.

Mathematics

Standards, Benchmarks and Indicators

Prekindergarten – Grade 2



Correlation of Benchmarks and Indicators

Prekindergarten — 2

NOTE: The term, *prekindergarten*, refers to all early learning experiences before kindergarten (e.g., nursery school, preschool, family care, etc.).

The number in parenthesis () corresponds to the numbered grade-level indicator found in section “Benchmarks and Indicators by Standard” of *Academic Content Standards: K-12 Mathematics* document.

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

A. Use place value concepts to represent whole numbers using numerals, words and physical models.

Prekindergarten Indicators	Kindergarten Indicators	Grade 1 Indicators	Grade 2 Indicators
<ul style="list-style-type: none"> • Represent quantity using invented forms (e.g., child’s marks to represent a quantity of objects). (8) • Write numerical representations (e.g., scribbles, reversals) or numerals in meaningful context (e.g., play situations). (9) • Identify and name numerals 0-9. (10) 	<ul style="list-style-type: none"> • Relate, read and write numerals for single-digit numbers (0 to 9). (5) 	<ul style="list-style-type: none"> • Use place value concepts to represent whole numbers using numerals, words, expanded notation and physical models with ones and tens. For example: <ol style="list-style-type: none"> a. Develop a system to group and count by twos, fives and tens. b. Identify patterns and groupings in a 100’s chart and relate to place value concepts. c. Recognize the first digit of a two-digit number as the most important to indicate size of a number and the nearness to 10 or 100. (5) • Read and write the numerals for numbers to 100. (3) 	<ul style="list-style-type: none"> • Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example: <ol style="list-style-type: none"> a. Recognize 10 can mean “10 ones” or a single entity (1 ten) through physical models and trading games. b. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each. (1)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

B. Recognize, classify, compare and order whole numbers.

Prekindergarten Indicators	Kindergarten Indicators	Grade 1 Indicators	Grade 2 Indicators
<ul style="list-style-type: none"> Compare and order whole numbers up to 5. (11) Compare sets of equal, more and fewer and use the language of comparison (e.g., equal, more and fewer). (6) 	<ul style="list-style-type: none"> Compare and order whole numbers up to 10. (1) Compare the number of objects in two or more sets when one set has one or two more, or one or two fewer objects. (7) Recognize the number or quantity of sets up to 5 without counting; e.g., recognize without counting the dot arrangement on a domino as 5. (13) 	<ul style="list-style-type: none"> Use ordinal numbers to order objects; e.g., first, second, third. (1) Recognize and generate equivalent forms for the same number using physical models, words and number expressions; e.g., concept of ten is described by “10 blocks,” full tens frame, numeral 10, $5+5$, $15 - 5$, one less than 11, my brother’s age. (2) Count forward to 100, count backwards from 100, and count forward or backward starting at any number between 1 and 100. (4) Demonstrate that equal means “the same as” using visual representations. (15) 	<ul style="list-style-type: none"> Use place value concepts to represent, compare and order whole numbers using physical models, numerals and words, with ones, tens and hundreds. For example: <ol style="list-style-type: none"> Recognize 10 can mean “10 ones” or a single entity (1 ten) through physical models and trading games. Read and write 3-digit numerals (e.g., 243 as two hundred forty three, 24 tens and 3 ones, or 2 hundreds and 43 ones, etc.) and construct models to represent each. (1) Recognize and classify numbers as even or odd. (2)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

C. Represent commonly used fractions using words and physical models.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Represent commonly used fractions using words and physical models for halves, thirds and fourths, recognizing fractions are represented by equal size parts of a whole and of a set of objects. (9)

Grade 2 Indicators

- Represent fractions (halves, thirds, fourths, sixths and eighths), using words, numerals and physical models. For example:
 - a. Recognize that a fractional part can mean different amounts depending on the original quantity.
 - b. Recognize that a fractional part of a rectangle does not have to be shaded with contiguous parts.
 - c. Identify and illustrate parts of a whole and parts of sets of objects.
 - d. Compare and order physical models of halves, thirds and fourths in relations to 0 and 1. (5)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

D. Determine the value of a collection of coins and dollar bills.

Prekindergarten Indicators

- Identify penny, nickel, dime and quarter and recognize that coins have different values. (12)

Kindergarten Indicators

- Identify and state the value of a penny, nickel and dime. (9)

Grade 1 Indicators

- Identify and state the value of a penny, nickel, dime, quarter and dollar. (6)
- Determine the value of a small collection of coins (with a total value up to one dollar) using 1 or 2 different type coins, including pennies, nickels, dimes and quarters. (7)

Grade 2 Indicators

- Represent and write the value of money using the ¢ sign and in decimal form when using the \$ sign. (4)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

E. Make change using coins for values up to one dollar.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Show different combinations of coins that have the same value. (8)

Grade 2 Indicators

- Count money and make change using coins and a dollar bill. (3)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

F. Count, using numerals and ordinal numbers.

Prekindergarten Indicators

- Touch objects and say the number names when counting in the context of daily activities and play (e.g., cookies on a plate, steps on a set of stairs). (2)
- Demonstrate one-to-one correspondence when counting objects (e.g., give one cookie to each child in group). (3)
- Count to 10 in the context of daily activities and play (e.g., number songs). (1)
- Determine “how many” in sets of 5 or fewer objects. (4)

Kindergarten Indicators

- Explain rules of counting, such as each object should be counted once and that order does not change the number. (2)
- Count to twenty; e.g., in play situations or while reading number books. (3)
- Determine “how many” in sets (groups) of 10 or fewer objects. (4)

Grade 1 Indicators

- Count forward to 100, count backwards from 100, and count forward or backward starting at any number between 1 and 100. (4)

Grade 2 Indicators

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

G. Model, represent and explain addition as combining sets and counting on.

Prekindergarten Indicators

- Group and regroup a given set in the context of daily activities and play (e.g., 5 blocks can be 2 blue and 3 green or 1 blue and 4 green). (7)
- Construct sets with more or fewer objects than a given set. (13)
- Count on (forward) using objects such as cards, number cubes or dominoes that have familiar dot patterns (e.g., when selecting 5 apples from a bag, takes out two and continues counting 3, 4, 5). (14)

Kindergarten Indicators

- Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green. (8)
- Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
 - a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
 - b. Count on (forward) and count back (backward) on a number line between 0 and 10. (10)

Grade 1 Indicators

- Model, represent and explain addition as combining sets (part + part = whole) and counting on. For example:
 - a. Model and explain addition using physical materials in contextual situations.
 - b. Draw pictures to model addition.
 - c. Write number sentences to represent addition.
 - d. Explain that adding two whole numbers yields a larger whole number. (10)
- Use conventional symbols to represent the operations of addition and subtraction. (12)

Grade 2 Indicators

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

H. Model, represent and explain subtraction as comparison, take-away and part-to-whole.

Prekindergarten Indicators

- Group and regroup a given set in the context of daily activities and play (e.g., 5 blocks can be 2 blue and 3 green or 1 blue and 4 green). (7)
- Construct sets with more or fewer objects than a given set. (13)
- Count on (forward) using objects such as cards, number cubes or dominoes that have familiar dot patterns (e.g., when selecting 5 apples from a bag, takes out two and continues counting 3, 4, 5). (14)

Kindergarten Indicators

- Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green. (8)
- Model and represent addition as combining sets and counting on, and subtraction as take-away and comparison. For example:
 - a. Combine and separate small sets of objects in contextual situations; e.g., add or subtract one, two, or another small amount.
 - b. Count on (forward) and count back (backward) on a number line between 0 and 10. (10)

Grade 1 Indicators

- Model, represent and explain subtraction as take-away and comparison. For example:
 - a. Model and explain subtraction using physical materials in contextual situations.
 - b. Draw pictures to model subtraction.
 - c. Write number sentences to represent subtraction.
 - d. Explain that subtraction of whole numbers yields an answer smaller than the original number. (11)
- Use conventional symbols to represent the operations of addition and subtraction. (12)

Grade 2 Indicators

- Model, represent and explain subtraction as comparison, take-away and part-to-whole; e.g., solve missing addend problems by counting up or subtracting, such as “I had six baseball cards, my sister gave me more, and I now have ten. How many did she give me?” can be represented as $6 + ? = 10$ or $10 - 6 = ?$ (6)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

I. Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting.

Prekindergarten Indicators

- Construct two sets of objects, each containing the same number of objects (e.g., 5 crayons and 5 blocks). (5)
- Join two sets of objects to make one large set in the context of daily routines and play (e.g., combining 2 bags of raisins, each containing 3 pieces; combining 2 groups of blocks, each containing 3 blocks). (15)

Kindergarten Indicators

- Construct multiple sets of objects each containing the same number of objects. (6)
- Demonstrate joining multiple groups of objects, each containing the same number of objects; e.g., combining 3 bags of candy, each containing 2 pieces. (11)

Grade 1 Indicators

- Model and represent multiplication as repeated addition and rectangular arrays in contextual situations; e.g., four people will be at my party and if I want to give 3 balloons to each person, how many balloons will I need to buy? (13)

Grade 2 Indicators

- Model, represent and explain multiplication as repeated addition, rectangular arrays and skip counting. (7)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

J. Model, represent and explain division as sharing equally, repeated subtraction and rectangular arrays.

Prekindergarten Indicators

- Distribute equally a set of objects into 2 or more smaller sets (e.g., shares 6 crackers with 3 friends equally). (16)

Kindergarten Indicators

- Partition or share a small set of objects into groups of equal size; e.g., sharing 6 stickers equally among 3 children. (12)

Grade 1 Indicators

- Model and represent division as sharing equally in contextual situations; e.g., sharing cookies. (14)

Grade 2 Indicators

- Model, represent and explain division as sharing equally and repeated subtraction. (8)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

K. Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions.

Prekindergarten Indicators

- Group and regroup a given set in the context of daily activities and play (e.g., 5 blocks can be 2 blue and 3 green or 1 blue and 4 green). (7)

Kindergarten Indicators

- Represent and use whole numbers in flexible ways, including relating, composing and decomposing numbers; e.g., 5 marbles can be 2 red and 3 green or 1 red and 4 green. (8)

Grade 1 Indicators

- Develop strategies for basic addition facts, such as:
 - a. counting all;
 - b. counting on;
 - c. one more, two more;
 - d. doubles;
 - e. doubles plus or minus one;
 - f. make ten;
 - g. using tens frames;
 - h. identity property (adding zero). (16)
- Develop strategies for basic subtraction facts, such as:
 - a. relating to addition (for example, think of $7 - 3 = ?$ as “3 plus ? equals 7”);
 - b. one less, two less;
 - c. all but one (for example, $8 - 7$, $5 - 4$);
 - d. using tens frames;
 - e. missing addends. (17)

Grade 2 Indicators

- Demonstrate fluency in addition facts with addends through 9 and corresponding subtractions; e.g., $9 + 9 = 18$, $18 - 9 = 9$. (10)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

L. Demonstrate fluency in adding and subtracting multiples of 10 and recognize combinations that make 10.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Develop strategies for basic addition facts, such as:
 - a. counting all;
 - b. counting on;
 - c. one more, two more;
 - d. doubles;
 - e. doubles plus or minus one;
 - f. make ten;
 - g. using tens frames;
 - h. identity property (adding zero). (16)
- Develop strategies for basic subtraction facts, such as:
 - a. relating to addition (for example, think of $7 - 3 = ?$ as “3 plus ? equals 7”);
 - b. one less, two less;
 - c. all but one (for example, $8 - 7$, $5 - 4$);
 - d. using tens frames;
 - e. missing addends. (17)

Grade 2 Indicators

- Add and subtract multiples of 10. (11)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Number, Number Sense and Operations Standard

Pre-K – 2 Benchmark

M. Add and subtract two-digit numbers with and without regrouping.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

Grade 2 Indicators

- Model and use the commutative property for addition. (9)
- Demonstrate multiple strategies for adding and subtracting 2- or 3-digit whole numbers, such as:
 - a. compatible numbers;
 - b. compensatory numbers;
 - c. informal use of commutative and associative properties of addition. (12)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Measurement Standard

Pre-K – 2 Benchmark

A. Explain the need for standard units of measure.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Recognize and explain the need for fixed units and tools for measuring length and weight; i.e., rulers and balance scales. (1)

Grade 2 Indicators

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Measurement Standard

Pre-K – 2 Benchmark

B. Select appropriate units for length, weight, volume (capacity) and time, using: 1) objects; i.e., non-standard units; 2) U.S. customary units: inch, foot, yard, ounce, pound, cup, quart, gallon, minute, hour, day, week and year; 3) metric units: centimeter, meter, gram and liter.

Prekindergarten Indicators

- Begin to identify and use language of units of time. For example:
 - a. Day, night, week;
 - b. Yesterday, today, tomorrow. (1)
- Recognize that various devices measure time (e.g., clock, timer, calendar). (2)
- Begin to use terms to compare the attributes of objects (e.g., bigger, smaller, lighter, heavier, taller, shorter, more and less). (4)
- Order a set of objects according to size, weight or length (e.g., cups of different sizes). (5)

Kindergarten Indicators

- Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days. (1)
- Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less. (2)

Grade 1 Indicators

Grade 2 Indicators

- Identify and select appropriate units of measure for:
 - a. length – centimeters, meters, inches, feet, or yards;
 - b. volume (capacity) – liters, cups, pints, or quarts;
 - c. weight – grams, ounces, or pounds;
 - d. time – hours, half-hours, quarter-hours, or minutes and time designations a.m. or p.m. (1)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Measurement Standard

Pre-K – 2 Benchmark

C. Develop common referents for units of measure for length, weight, volume (capacity) and time to make comparisons and estimates.

Prekindergarten Indicators

- Begin to identify and use language of units of time. For example:
 - a. Day, night, week;
 - b. Yesterday, today, tomorrow. (1)
- Recognize that various devices measure time (e.g., clock, timer, calendar). (2)
- Begin to use terms to compare the attributes of objects (e.g., bigger, smaller, lighter, heavier, taller, shorter, more and less). (4)
- Order a set of objects according to size, weight or length (e.g., cups of different sizes). (5)
- Sequence or order events in the context of daily activities and play (e.g., wash your hands before and after snacks, who's next for the computer). (3)

Kindergarten Indicators

- Identify units of time (day, week, month, year) and compare calendar elements; e.g., weeks are longer than days. (1)
- Compare and order objects of different lengths, areas, weights and capacities; and use relative terms, such as longer, shorter, bigger, smaller, heavier, lighter, more and less. (2)
- Order events based on time. For example:
 - a. activities that take a long or short time;
 - b. review what we do first, next, last;
 - c. recall what we did or plan to do yesterday, today, tomorrow. (4)

Grade 1 Indicators

- Tell time to the hour and half hour on digital and analog (dial) timepieces. (2)
- Order a sequence of events with respect to time; e.g., summer, fall, winter and spring; morning, afternoon and night. (3)

Grade 2 Indicators

- Establish personal or common referents for units of measure to make estimates and comparisons; e.g., the width of a finger is a centimeter, a large bottle of soda pop is 2 liters, a small paper clip weighs about one gram. (2)
- Tell time to the nearest minute interval on digital and to the nearest 5 minute interval on analog (dial) timepieces. (4)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Measurement Standard

Pre-K – 2 Benchmark

D. Apply measurement techniques to measure length, weight and volume (capacity).

Prekindergarten Indicators

- Measure length and volume (capacity) using non-standard units of measure (e.g., how many paper clips long is a pencil, how many small containers it takes to fill one big container using sand, rice or beans). (6)

Kindergarten Indicators

- Measure length and volume (capacity) using uniform objects in the environment. For example, find:
 - a. how many paper clips long is a pencil;
 - b. how many small containers it takes to fill one big container using sand, rice, beans. (3)

Grade 1 Indicators

- Estimate and measure weight using non-standard units; e.g., blocks of uniform size. (4)
- Estimate and measure lengths using non-standard and standard units; i.e., centimeters, inches and feet. (5)

Grade 2 Indicators

- Estimate and measure the length and weight of common objects, using metric and U.S. customary units, accurate to the nearest unit. (5)
- Select and use appropriate measurement tools; e.g., a ruler to draw a segment 3 inches long, a measuring cup to place 2 cups of rice in a bowl, a scale to weigh 50 grams of candy. (6)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Measurement Standard

Pre-K – 2 Benchmark

E. Recognize that using different units of measurement will yield different numbers for the same measurement.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

Grade 2 Indicators

- Describe and compare the relationships among units of measure, such as centimeters and meters; inches, feet and yards; cups, pints and quarts; ounces and pounds; and hours, half-hours, and quarter-hours; e.g., how many inches in a foot? (3)
- Make and test predictions about measurements, using different units to measure the same length or volume. (7)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

A. Describe and create plane figures: circle, rectangle, square, triangle, hexagon, trapezoid, parallelogram and rhombus, and identify them in the environment.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Create new shapes by combining or cutting apart existing shapes. (2)
- Identify the shapes of the faces of three-dimensional objects. (3)

Grade 2 Indicators

- Identify, describe, compare, and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the numbers of faces, edges, or vertices. (1)
- Predict what new shapes will be formed by combining or cutting apart existing shapes. (2)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

B. Describe solid objects: cube, rectangular prism, sphere, cylinder, cone and pyramid, and identify them in the environment.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Identify the shapes of the faces of three-dimensional objects. (3)

Grade 2 Indicators

- Identify, describe, compare, and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the numbers of faces, edges, or vertices. (1)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

C. Sort and compare two-dimensional figures and three-dimensional objects according to their characteristics and properties.

Prekindergarten Indicators

- Match identical two- and three-dimensional objects found in the environment and in play situations (e.g., 2 squares of same size, 2 stop signs). (1)
- Sort and classify similar two- and three-dimensional objects in the environment and play situations (e.g., paper shapes, 2 balls of different size). (2)
- Identify, name, create and describe common two-dimensional shapes in the environment and play situations (e.g., circles, triangles, rectangles and squares). (3)
- Identify, name and describe three-dimensional objects using the child's own vocabulary (e.g., sphere-“ball,” cube-“box,” cylinder-“can” or “tube,” cone-“ice cream cone”). (4)

Kindergarten Indicators

- Identify and sort two-dimensional shapes and three-dimensional objects. For example:
 - a. Identify and describe two-dimensional figures and three-dimensional objects from the environment using the child's own vocabulary.
 - b. Sort shapes and objects into groups based on student-defined categories.
 - c. Select all shapes or objects of one type from a group.
 - d. Build two-dimensional figures using paper shapes or tangrams; build simple three-dimensional objects using blocks. (1)

Grade 1 Indicators

- Identify, compare, and sort two-dimensional shapes; i.e., square, circle, ellipse, triangle, rectangle, rhombus, trapezoid, parallelogram, pentagon, and hexagon. For example:
 - a. Recognize and identify triangles and rhombuses independent of position, shape or size;
 - b. Describe two-dimensional shapes using attributes such as number of sides and number of vertices (corners, or angles). (1)

Grade 2 Indicators

- Identify, describe, compare, and sort three-dimensional objects (i.e., cubes, spheres, prisms, cones, cylinders and pyramids) according to the shape of the faces or the number of faces, edges, or vertices. (1)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

D. Identify, explain and model (superposition, copying) the concept of shapes being congruent and similar.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Copy figures and draw simple two-dimensional shapes from memory. (5)

Grade 2 Indicators

- Identify and determine whether two-dimensional shapes are congruent (same shape and size) or similar (same shape different size) by copying or using superposition (lay one thing on top of another). (4)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

E. Recognize two- and three-dimensional objects from different positions.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Copy figures and draw simple two-dimensional shapes from memory. (5)

Grade 2 Indicators

- Recognize two-dimensional shapes and three-dimensional objects from different positions. (3)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

F. Describe location, using comparative (before, after), directional (above, below) and positional (first, last) words.

Prekindergarten Indicators

- Demonstrate and begin to use the language of the relative position of objects in the environment and play situations (e.g., up, down, over, under, top, bottom, inside, outside, in front, behind, between, beside, next to, right side up and upside down). (5)

Kindergarten Indicators

- Name and demonstrate the relative position of objects as follows:
 - a. place objects over, under, inside, outside, on, beside, between, above, below, on top of, upside-down, behind, in back of, in front of;
 - b. describe placement of objects with terms such as on, inside, outside, above, below, over, under, beside, between, in front of, behind. (2)

Grade 1 Indicators

- Extend the use of location words to include distance (near, far, close to) and directional words (left, right). (4)

Grade 2 Indicators

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Geometry and Spatial Sense Standard

Pre-K – 2 Benchmark

G. Identify and draw figures with line symmetry.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Copy figures and draw simple two-dimensional shapes from memory. (5)

Grade 2 Indicators

- Create and identify two-dimensional figures with line symmetry; e.g., what letter shapes, logos, polygons are symmetrical? (5)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

A. Sort, classify and order objects by size, number and other properties, and describe the attributes used.

Prekindergarten Indicators

- Sort, order and classify objects by one attribute (e.g., size, color, shape, use). (1)

Kindergarten Indicators

- Sort, classify and order objects by size, number and other properties. For example:
 - Identify how objects are alike and different.
 - Order three events or objects according to a given attribute, such as time or size.
 - Recognize and explain how objects can be classified in more than one way.
 - Identify what attribute was used to sort groups of objects that have already been sorted. (1)

Grade 1 Indicators

- Sort, classify and order objects by two or more attributes, such as color and shape, and explain how objects were sorted. (1)

Grade 2 Indicators

-

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

B. Extend sequences of sounds and shapes or simple number patterns, and create and record similar patterns.

Prekindergarten Indicators

- Identify, copy, extend and create simple patterns or sequences of sounds, shapes and motions in the context of daily activities and play (e.g., creates red, blue, red, blue pattern with blocks). (2)

Kindergarten Indicators

- Identify, create, extend and copy sequences of sounds (such as musical notes), shapes (such as buttons, leaves or blocks), motions (such as hops or skips), and numbers from 1 to 10. (2)

Grade 1 Indicators

- Extend sequences of sounds, shapes or simple number patterns, and create and record similar patterns. For example:
 - a. Analyze and describe patterns with multiple attributes using numbers and shapes; e.g., AA, B, aa, b, AA, B, aa, b,....
 - b. Continue repeating and growing patterns with materials, pictures and geometric items; e.g., XO, XOO, XOOO, XOOOO. (2)

Grade 2 Indicators

- Extend simple number patterns (both repeating and growing patterns), and create similar patterns using different objects, such as using physical materials or shapes to represent numerical patterns. (1)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

C. Create and extend patterns and describe the rule in words.

Prekindergarten Indicators

- Sequence or order events in the context of daily activities and play (e.g., wash your hands before and after snacks, who's next for the computer). (3 under measurement)

Kindergarten Indicators

- Describe orally the pattern of a given sequence. (3)

Grade 1 Indicators

- Describe orally the basic unit or general plan of a repeating or growing pattern. (3)

Grade 2 Indicators

- Use patterns to make generalizations and predictions; e.g., determine a missing element in a pattern. (2)
- Create new patterns with consistent rules or plans, and describe the rule or general plan of existing patterns. (3)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

D. Model problem situations using objects, pictures, tables, numbers, letters and other symbols.

Prekindergarten Indicators

- Use play, physical materials or drawings to model a simple problem (e.g., There are 6 cookies to be shared by 3 children. How many cookies can each child receive?). (3)

Kindergarten Indicators

- Model a problem situation using physical materials. (4)

Grade 1 Indicators

- Describe orally and model a problem situation using words, objects or number phrase or sentence. (5)

Grade 2 Indicators

- Use objects, pictures, numbers and other symbols to represent a problem situation. (4)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

E. Solve open sentences and explain strategies.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Solve open sentences by representing an expression in more than one way using the commutative property; e.g., $4 + 5 = 5 + 4$ or the number of blue balls plus red balls is the same as the number of red balls plus blue balls ($R+B=B+R$). (4)

Grade 2 Indicators

- Understand equivalence and extend the concept to situations involving symbols: e.g., $4 + 5 = 9$ and $9 = 4 + 5$ and $4 + 5 = 3 + 6 = \Delta + \dots$ (5)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

F. Represent an unknown quantity as a variable using a symbol such as \square , Δ , \circ .

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

Grade 2 Indicators

- Use symbols to represent unknown quantities and identify values for symbols in an expression or equation using addition and subtraction; e.g.,
 $\square + \circ = 10$, $\Delta - 2 = 4$.
(6)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Patterns, Functions and Algebra Standard

Pre-K – 2 Benchmark

G. Describe and compare qualitative and quantitative change.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

Grade 2 Indicators

- Describe qualitative and quantitative changes, especially those involving addition and subtraction; e.g., a student growing taller versus a student growing two inches in one year. (7)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Data Analysis and Probability Standard

Pre-K – 2 Benchmark

A. Pose questions and gather data about everyday situations and familiar objects.

Prekindergarten Indicators

- Gather, sort and compare objects by similarities and differences in the context of daily activities and play (e.g., leaves, nuts, socks). (1)

Kindergarten Indicators

- Gather and sort data in response to questions posed by teacher and students; e.g., how many sisters and brothers, what color shoes. (1)

Grade 1 Indicators

- Construct a question that can be answered by using information from a graph. (5)

Grade 2 Indicators

- Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs. (1)
- Recognize that data may vary from one population to another; e.g., favorite TV shows of students and of parents. (6)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Data Analysis and Probability Standard

Pre-K – 2 Benchmark

B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.

Prekindergarten Indicators

- Place information or objects in a floor or table graph according to one attribute (e.g., size, color, shape or quantity). (2)
- Select the category or categories that have the most or fewest objects in a floor or table graph (e.g., favorite ice cream). (3)

Kindergarten Indicators

- Arrange objects in a floor or table graph according to attributes, such as use, size, color, or shape. (2)
- Select the category or categories that have the most or fewest objects in a floor or table graph. (3)

Grade 1 Indicators

- Identify multiple categories for sorting data. (10)
- Collect and organize data into charts using tally marks. (2)
- Arrange five objects by an attribute, such as size or weight, and identify the ordinal position of each object. (6)
- Answer questions about the number of objects represented in a picture graph, bar graph or table graph; e.g., category with most, how many more in a category compared to another, how many altogether in two categories. (7)

Grade 2 Indicators

- Pose questions, use observations, interviews and surveys to collect data, and organize data in charts, picture graphs and bar graphs. (1)
- Write a few sentences to describe and compare categories of data represented in a chart or graph, and make statements about the data as a whole. (4)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Data Analysis and Probability Standard

Pre-K – 2 Benchmark

C. Represent data using objects, picture graphs and bar graphs.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Display data in picture graphs with units of 1 and bar graphs with intervals of 1. (3)
- Read and interpret charts, picture graphs and bar graphs as sources of information to identify main ideas, draw conclusions, and make predictions. (4)

Grade 2 Indicators

- Read, interpret and make comparisons and predictions from data represented in charts, line plots, picture graphs and bar graphs. (2)
- Read and construct simple timelines to sequence events. (3)
- Identify untrue or inappropriate statements about a given set of data. (5)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Data Analysis and Probability Standard

Pre-K – 2 Benchmark

D. Describe the probability of chance events as more, less or equally likely to occur.

Prekindergarten Indicators

There are no indicators age appropriate for this level.

Kindergarten Indicators

There are no indicators age appropriate for this level.

Grade 1 Indicators

- Describe the likelihood of simple events as possible/impossible and more likely/less likely; e.g., when using spinners or number cubes in classroom activities. (8)

Grade 2 Indicators

- List some of the possible outcomes of a simple experiment, and predict whether given outcomes are more, less or equally likely to occur. (7)
- Use physical models and pictures to represent possible arrangements of 2 or 3 objects. (8)

Correlation of Benchmarks and Indicators

Prekindergarten — 2

Mathematical Processes Standard

Pre-K – 2 Benchmarks

- A. Use a variety of strategies to understand problem situations; e.g., discussing with peers, stating problems in own words, modeling problems with diagrams or physical materials, identifying a pattern.
- B. Identify and restate in own words the question or problem and the information needed to solve the problem.
- C. Generate alternative strategies to solve problems.
- D. Evaluate the reasonableness of predictions, estimations and solutions.
- E. Explain to others how a problem was solved.
- F. Draw pictures and use physical models to represent problem situations and solutions.
- G. Use invented and conventional symbols and common language to describe a problem situation and solution.
- H. Recognize the mathematical meaning of common words and phrases, and relate everyday language to mathematical language and symbols.
- I. Communicate mathematical thinking by using everyday language and appropriate mathematical language.

Note: The benchmarks for mathematical processes articulate what children should demonstrate in problem solving, representation, communication, reasoning and connections at key points in their mathematics program. Specific grade-level indicators have not been included for the mathematical processes standard because content and processes have been embedded within the grade-level indicators for the five content standards.

Mathematics

Instructional Commentary





Mathematics in the Early Years



Research indicates that long before entering school, children spontaneously explore and intuitively use number concepts. Mathematical foundations begin as children participate in activities such as giving a cracker to each peer at the table, pouring water from one container into another, putting all the big buttons in one pile and the smaller ones in another, or clapping a rhythmic pattern to words or a song. Everyday experiences such as these provide the context for preschool children to progress in mathematics.

Early childhood educators can foster children’s mathematical development by providing environments rich in language, where thinking is encouraged and exploring is supported. The preschool environment should include concrete mathematics manipulatives and materials for exploration and should allow children to learn mathematics through real situations and playful activities that encourage counting, measuring, patterning and mathematical problem-solving.

Young children’s knowledge of mathematics is constructed over time. Teachers should ensure that mathematics experiences are woven throughout the curriculum, follow logical sequences, and support depth and focus of meaning. Besides embedding experiences across the curriculum, high quality early mathematics teaching requires carefully planned experiences that focus children’s attention on a particular mathematical concept. The design of appropriate and effective early mathematics programs requires educators to stay focused on the “big ideas” of mathematics and on the sequences and connections among these ideas.

Both national standards and Ohio’s early learning content standards reflect core ideas in five major content areas: number, number sense and operations; measurement; geometry and spatial sense; algebra and patterning; and data analysis. Within these content areas, appropriate mathematics programs for young children provide environments and experiences that encourage exploring and understanding the following “big ideas”: number sense and counting, including one-to-one correspondence; classifying; comparing one part to another and a part to the whole; grouping and sharing; measurement; shapes and space; patterning and ordering; and mathematical language development within natural contexts. Problem solving and reasoning are the heart of mathematics. Thus, children are encouraged to observe, question, collect information, communicate ideas, make connections and representations, and reflect on their mathematical experiences within problem-solving situations. Through high quality educational environments and intentional experiences, educators provide children with a solid cognitive foundation in mathematics.



Understanding Mathematical Processes

It is important for teachers and curriculum leaders implementing standards-based instruction in Ohio's classrooms to understand mathematical content and mathematical processes when making instructional decisions. Students need to learn and apply the content and skills that form the basis of mathematics while using mathematical processes. Content and processes are not isolated in mathematics, but are interconnected parts of a whole.

The mathematics standards are written as five content standards and one process standard. Within the standards documents, the mathematical processes are specified only through benchmark statements. This is intentional, since good instruction consists of teaching mathematical content through mathematical processes.

The processes involved in learning mathematics are especially important to teachers of young children. Throughout the five content standards, children should be guided to use mathematical processes of problem solving, reasoning, communicating, representing and connecting. The following describes each mathematical process and offers ideas for supporting learning within early childhood mathematics education.

Problem Solving: All young children solve problems. An effective problem solver perseveres, is focused, takes reasonable risks, remains flexible, tests hypotheses, tries alternatives and exhibits self-regulation. While young children have far to go before they reach the level of mature problem solvers, they are already learning important lessons (Copley, 2000).

A key teaching role that promotes problem solving is to provide the environment, materials and experiences for solving problems. Early childhood educators should design the learning environment so it offers interesting problems and an array of materials for children to use in solving them, such as counters, geometric shapes, blocks and puzzles.

Informal opportunities for problem solving occur daily in the early childhood classroom. Instead of offering solutions, teachers and adults can help children state problems, provide time to listen and talk about the problems, and connect some of the problems occurring in their lives to mathematics learning. Of equal importance, young children learn powerful lessons when teachers model problem-solving strategies, convey the joys of finding solutions and acknowledge the frustrations during challenging moments (Copley, 2000).

Reasoning: Reasoning opportunities are abundant in the life of a young child. In early childhood settings, children classify and label sets of objects, play games, solve problems, and observe and listen to others as they think.

Teachers and adults support children's reasoning by providing experiences that help them develop clarity and accuracy in their thinking. To give children familiarity with the language of logic and reasoning, teachers should model and use phrases such as *or*, *not*, *if*, *then*, *because*, *some*, *all*, *never* and *probably*. For example, a teacher could say, "You may choose apple juice or milk to drink with your crackers," or, "All of you have families, and some of you have brothers in your families" (Copley, 2000).



Teachers should encourage children to make, investigate and justify hypotheses. Adults should ask questions that require investigation and reasoning, such as, “Are you sure?, How do you know?, What would happen if...?, I wonder how this could be changed?, I wonder why...?”. In addition, teachers thinking out loud and expressing their own thoughts about problems can facilitate children’s thinking. Finally, and most importantly, teachers must listen to children’s own reasoning. Only when a teacher knows and understands a young child’s justifications can he or she best encourage more sophisticated and reasonable hypotheses.

Communicating: To communicate mathematical ideas, children must articulate, organize, clarify and consolidate their thinking. As they struggle to capture their ideas and processes in words or through other modes of communication, young children become more aware of what they know and how they solve problems.

Children should have an opportunity to talk with and to listen to their peers and use a variety of verbal and nonverbal means to communicate their mathematical ideas to others. They manipulate objects, use fingers, draw pictures and create other ways to show what they know and mean. They also learn to explain their solutions in writing, use diagrams and charts, and express ideas with mathematics symbols (Copley, 2000).

To promote children’s development in mathematics communication, educators find it helpful to verbalize and restate concepts and processes. They ask questions, describe what they observe the child doing, and listen attentively to young children’s communication efforts. Through listening, teachers gain an important window into a child’s mind.

Representing: Representations for numbers, operations, patterns and geometric concepts are all essential for mathematical understanding. The young child’s use of representations includes those that the teacher or adult introduced as well as those that the child created.

To encourage flexibility in mathematical representations, early childhood teachers should introduce children to a wide range of representations, such as pictorial (drawings, maps); graphical (bar graphs created from stacked objects, timelines, pictographs); and symbolic (tables). Teachers and parents often introduce children to common forms of numerical representations, such as tally marks, using fingers, pictures of objects, and dots used in ten frames.

Providing meaningful context that encourages children to represent and communicate their understanding is part of the teacher’s role. Children should learn that representation helps them remember what they did and explain their thinking and reasoning. In addition, adults should also ask children to verbally or concretely represent a concept or to use their words and objects to communicate their ideas. Representation is closely linked to communication, each informing and supporting the other (Copley, 2000).

Connecting: According to *Principles and Standards for School Mathematics* (NCTM, 2000), “The most important connection for early childhood mathematics development is between the intuitive, informal mathematics that students have learned through their own experiences and the mathematics they are learning in school. A student’s ability to experience mathematics as a meaningful endeavor rests on these connections. Understanding connections helps children realize the beauty of mathematics and its function as a means of more clearly observing, representing, and interpreting the world around them.”

Teachers play a critical role in helping young children connect mathematics to other parts of the curriculum and to their world. An infinite number of connections can be made quite naturally between mathematics and literature, language, art, construction, science, physical movement and music. Children tapping and repeating rhythmic patterns in music, counting letters and constructing a class graph to represent the length of names, and creating sketches of block constructions are but a few. It is important that these connections be natural and not forced or contrived (Copley, 2000).

When children represent ideas mathematically or connect representations to mathematics, their understanding of math is enhanced. Young children convey their mathematical ideas in a variety of ways. They use concrete objects, including their own fingers; draw; produce diagrams; and make tallies, symbols and markings of various types.

As previously stated, the five mathematical processes are used in all content standards for mathematics and should be incorporated into other curriculum content and as part of instructional strategies and assessment procedures as relevant. The following examples of prekindergarten indicators show how process skills are embedded within the mathematics content standards.

Number, Number Sense and Operations

Prekindergarten Indicator

8. *Represent quantity using invented forms (e.g., a child's marks to represent a quantity of objects).*

- *Processes: representing, connecting*

Measurement

Prekindergarten Indicator

4. *Begin using terms to compare the attributes of objects (e.g., bigger, smaller, lighter, heavier, taller, shorter, more, less).*

- *Processes: communicating, reasoning, connecting*

Geometry and Spatial Sense

Prekindergarten Indicator

3. *Identify, name, create and describe common two-dimensional shapes found in the environment and in play situations (e.g., circles, triangles, rectangles and squares).*

- *Processes: reasoning, communicating, representing, connecting*

Patterns, Functions and Algebra

Prekindergarten Indicator

3. Use play, physical materials or drawings to model a simple problem (e.g., Six cookies are to be shared by three children. How many cookies can each child receive?).

- Processes: problem solving, representing, connecting, reasoning

Data Analysis and Probability

Prekindergarten Indicator

1. Gather, sort and compare objects according to their similarities and differences in the context of daily activities and play (e.g., leaves, nuts, socks).

- Processes: problem solving, reasoning, communicating, connecting

While content represents the “what” of early childhood mathematics education, the processes represent the “how,” making it possible for children to acquire content knowledge. Children’s development and use of these five processes are among the most long-lasting and important achievements of mathematics education. Experiences and intuitive ideas become truly mathematical as young children reflect on them, represent them in various ways and connect them to other ideas (NCTM, 2000).

When subject matter is dynamic, intellectually intriguing and personal — when it bestows power to the learner — the ‘details’ also become important and memorable.

– Carol Ann Tomlinson, 1999



Planning for Instruction



The vignette and lesson presented in this section show examples of classroom implementation of Ohio's early learning content standards for mathematics within the context of daily routines and activities. They illustrate how two teachers designed learning experiences to help children develop the knowledge and abilities identified in the standards, benchmarks and indicators. These examples can serve as starting points for discussion about instructional planning and implementation of the standards. In successful early childhood classrooms, the curriculum will be mapped to the standards and each experience will address many indicators identified across the mathematics standards and other content areas, thus integrating learning opportunities for children.

Benchmarks in each standard provide the goals for what students should know and be able to do by the time they reach the end of a grade-level band. While engaging children in these early mathematics experiences, the early childhood teacher uses the prekindergarten indicators as checkpoints for the specific knowledge and skills young children should demonstrate as a result of their learning experiences and intentional teaching. Early childhood educators should review the benchmarks to determine what children should be able to do by the end of their second grade year, giving further appreciation of their role in the continuum of children's education.

The following lesson serves as a suggestion. It may need to be modified to follow children's interests and serve individual educational needs. The context for the experience might be large group or small-group time following the shared reading of the book, *I Went Walking* by Sue Williams (Harcourt Brace Jovanovich, 1989). The primary focus of the experience will be on the data analysis and probability standard. However, as the teacher strives to extend children's learning, he or she also addresses other standards and indicators within this planned experience.

Graphing Shoe Fasteners

Standard: Data Analysis and Probability

Pre-K-2

- Benchmarks:**
- A. Pose questions and gather data about everyday situations and familiar objects.
 - B. Sort and classify objects by attributes, and organize data into categories in a simple table or chart.

- Indicators:**
- 1. Gather, sort and compare objects by similarities and differences in the context of daily activities and play (e.g., leaves, nuts, socks).
 - 2. Place information or objects in a floor or table graph according to one attribute (e.g., size, color, shape or quantity).
 - 3. Select the category or categories that have the most or fewest objects in a floor or table graph (e.g., favorite ice cream).

Purpose of activity:

Due to its abstract nature, graphing is often a difficult concept for young children to understand and use. Young children need multiple exposures, concrete examples and active participation in creating class graphs to represent and organize the sorting and categorizing of everyday objects in their environments. The following activity is a good introduction to graphs based on children's interests. It engages children in creating a table graph to represent what they know about fasteners on classmates' and their shoes.

Materials needed:

- Poster board or paper 22 by 16 inches (or a size large enough to accommodate all of the children's decisions), laminated or covered with clear contact paper
- A small buckle, a shoelace bow and a piece of hook and loop fastener, to identify each column
- A name card or tag for each child
- Tape to attach name tags to the chart

Procedure:

- Create a table graph with four columns, each column labeled by the following word(s) and the shoe fastener attached to column heading:
 - * buckle
 - * shoelaces
 - * hook and loop
 - * no fasteners
- Read and discuss a familiar children's book to serve as catalyst to the activity (e.g., *I Went Walking* by Sue Williams).
- Engage children in examining and discussing the similarities and differences in children's shoes, including fasteners.
- Introduce the concept of a graph as a way to visually sort and categorize their discoveries.
- Model the placement of the teacher's name within the column corresponding to the type of faster on his or her shoes.
- Invite the children to use a name card they place in the column to record their response to shoe fastener type.
- As children gather, sort and compare shoe fasteners, discuss rationale for their placement of name cards within columns.
- Following the recordings of children, invite children to count the numbers of cards within each column and analyze the information using question prompts such as:
 - * What is the most common type of shoe fastener in our class?
 - * How many people have shoelaces? Hook and loop tape? Buckles?
 - * How many more people have laces than have buckles?

- Look for the following response strategies used by children:
 - * Counting to quantify how many people have each type of shoe fastener.
 - * Comparing column heights to determine whether there are more, fewer, or the same number of name tags for each type of shoe fastener.
 - * Possibly subtracting to determine how many more name tags one column has than another.

Activity extensions:

- For some children, information representation may need to be more concrete. Consider placement of each child's actual shoe in the column to represent shoe fastener type; then move to the name card (picture/symbol representation) as the means to record the type.
- Read and discuss other books about shoes (e.g., *New Blue Shoes* by Eve Rice, *Shoes* by Elizabeth Winthrop)
- Create a shoe collection.
- Try shoe printing using small key chain shoes or novelty shoes.
- Make footprints by painting the children's feet and allowing them to walk along a strip of paper.
- Use the children's feet as a means to measure lengths or distance (e.g., length of road made in block area).
- Set up a shoe store in the dramatic play area.

Note: As a result of this experience, the following standard and prekindergarten indicators also are addressed:

Standard: Number, Number Sense and Operations

PreK-2

Benchmark: F. Count, using numerals and ordinal numbers.

- Indicators:**
2. Touch objects and say the number names when counting in the context of daily activities and play (e.g., cookies on a plate, steps on a set of stairs).
 3. Demonstrate one-to-one correspondence when counting objects (e.g., give one cookie to each child in group).

Opportunities to emphasize number and operations concepts are abundant in early childhood settings. In the following vignette, a preschool teacher uses an opportunity during her brief interaction with young children to match classroom instruction with the number, number sense and operations standard within Ohio's early learning content standards for mathematics.

In her prekindergarten class, Mrs. Armstrong observes four children in the dramatic play center. “Brother” Justin is setting the table for his pretend mommy, daddy, brother and company. Samuel, as the daddy, is reading the newspaper upside down and “Mommy” Amanda is cooking the meal. Nakita is watching, but is not involved in the dramatic play. There is one place setting each for the mommy, daddy, brother, and company; however, the company place setting includes all of the extra dishes stacked in a pile. To join the children’s play, Mrs. Armstrong asks, “Could I please be the company? And how about Nakita?” After Justin and Amanda agree, Mrs. Armstrong addresses Nakita, “Oooh, this meal looks great. I hope they have enough dishes, spoons and cups for both of us.”

Immediately, Justin and Amanda begin to match dishes with spoons and cups and to make two places for the company. They quickly notice that there are more than enough plates for the company, but not enough cups. “Daddy” Samuel is sent to the store to buy more — “just one more cup,” exclaims Justin. When Samuel returns with one more cup from the classroom sink, the imaginary meal is served and enjoyed by all.

The preceding lesson and vignette are but two examples of early childhood educators’ plans for and implementation of standards-based teaching and learning. Academic content standards serve to drive and inform curriculum, instruction and appropriate learning environments which are the contexts for planning and implementing developmentally appropriate and effective teaching practices to support the development of *all* young children in the area of mathematics.

Guidance for Early Learning Content Standards Implementation is an essential companion tool for educators of children ages 3 through 5. This document provides assistance in the design and implementation of meaningful curricula — intentional early learning experiences and practices aligned to standards-based indicators. The list of strategies and ideas to support young learners serves as a starting point for thoughtful curriculum design and teaching practices. Access this document online at <http://www.ode.state.oh.us>, keyword search: *guidance early learning*, or contact the Documents Resource Center of the Ohio Department of Education at (614) 728-3471 or toll-free at 1-877-644-6338.

The key to planning an integrated child-centered curriculum is balance — a balance among large group, small group and individual activities, a balance in curriculum and content areas, and a balance between teacher-directed and child-initiated experiences.

– Susan Schwartz and Mindy Pollishuke, 1991

Mathematics

Glossary





algebra – Most simply stated in an elementary setting, algebra is the study of generalizations of arithmetic. For example, a child may figure out that if two consecutive numbers are added together, the sum is adding the lower number twice and then adding one more number:

$$4 + 5 = 4 + (4 + 1) = (4 + 4) + 1; 6 + 7 = (6 + 6) + 1.$$

array – An orderly arrangement of objects in rows and columns.

attribute – A qualitative characteristic. Students are asked to group objects according to such attributes as color, size, shape and other identifiable characteristics.

bar graph – A graph in which the length of a bar (rectangle) is used to represent a numerical amount. A bar graph has spaces between the bars; a histogram does not have spaces.

concept – An underlying principle behind an idea.

concrete materials – Materials used to allow students hands-on experience with mathematics. Examples include counters and markers, also called manipulative materials.

cone – A three-dimensional figure whose base is a circle and whose sides taper to a point.

counting – At this level, children understand that the last number they count represents the total. They can make an equivalent set by counting the items in the sample set and then counting the same number of objects for the new set.

cube – A geometric solid with six square faces, each perpendicular to those adjoining it. More specifically, a rectangular solid with length, width and height all being equal is a cube.

cylinder – A three-dimensional figure with two congruent bases in parallel planes. The bases are generally circular, but this need not be so. Common shape is a soup can.

decomposing – The process of breaking a number into smaller units to simplify problem solving (e.g., 15 can be 10 + 5 or 10 can be 6 + 4).

floor or table graph – A graph made of actual objects arranged in categories on a tabletop or on the floor.

geometric figures – Polygons, circles and related three-figures dimensional solids.

invented – A child might write T4 to stand for 24 or npp to show she has a nickel and two pennies. Invented representation gives a child a format in which to communicate his or her understanding in a nonstandard format. We do not teach invented representation, but we appreciate a child's desire to communicate in ways that are personally meaningful.

multiple attributes – When objects have more than one characteristic, such as size, shape or color.

nonstandard measure – Using a measuring device that is not a widely accepted tool. For example, a measurement of six index fingers for length or 25 popped corn kernels for volume. This method is generally used as an approximation strategy.

one-to-one correspondence – At this stage, children attempt to make an equivalent set by taking an object for each object in the original set. They may align the two sets in rows and check to see if each object in the first set is opposite one object in the second set. In other words, when each element in a set is matched with exactly one element in a second set, and each element in the second is matched with exactly one element in the first set.

order – The arrangement of objects by a specific attribute.

ordinal position – Identifies the position of an object in a sequence, (e.g., first, second, third).

pattern – A pattern is something that repeats.

pattern blocks – Manipulative sets consisting of plastic or wooden triangles, squares, trapezoids, rhombuses and hexagons that can be used to show relationships between geometric figures, fractional parts and patterns.

physical strategies – A problem-solving method that incorporates building a model of the situation, acting out the problem narrative or using concrete materials.

pictograph – A type of graph which uses symbols to represent a quantity of objects or persons. The value of each symbol is shown in a key which appears on the graph.

representational graph – Pictures of real objects are placed on a wall or chalkboard.

relative position – Position of an object in relation to another (i.e., above, below, under, beside, before, after).

sequence – An ordered set of objects or numbers.

sphere – A three-dimensional surface where all points on the sphere are equally distant from a given point, called the center.

square – A rectangle with four equal sides. A rhombus with four right angles.

standard – The use of universally accepted units for measure.

stable order counting – Saying the words for numbers in a correct, consistent order when counting.

two-dimensional figures – A shape that has two dimensions, usually described in terms of length and breadth or length and height.

Venn Diagram – A diagram that is used to show relationships between sets.

Visual estimation or approximation – Children at this level quantify perceptually. They may make a large or small pile of objects to approximate the amount in a sample group.

weight – The physical volume or bulk of a solid body.

Active learning classrooms, based on principles of child development, produce long term gains in general intellectual growth, social and emotions skills, and life coping abilities.

– Author Unknown



Purposeful learning happens when teachers pose problems of relevance, provide open-ended opportunities for exploration, value children's suppositions, and structure learning around primary concepts within an integrated curriculum.

– Author Unknown

Mathematics

Resources





These resources can be used to aid in understanding content standards and to begin implementing standards-based instruction and assessment. The Instructional Management System (IMS) developed by the Ohio Department of Education provides opportunities to explore best practices, research-based instruction and effective lessons and strategies for all children and students. Access this resource online at <http://www.ode.state.oh.us>, keyword search: *Ohio's Instructional Management System*.

Instructional Resources

Resources listed in this section provide information for educators seeking practical and creative ways to implement the standards-based instruction.

Instructional Resources on the Internet

- **ETech Online Parent Modules** – The Ohio Department of Education and eTech Ohio have worked together to produce a series of fun and practical online resources for families, titled Learning and Growing Together. The resources include information about young children's development, scenarios that demonstrate ways to enhance early learning for reading, writing and mathematics, and additional activities and resources for families to support their young children's development. All resources shared are correlated with Ohio's early learning content standards. <http://www.ode.state.oh.us>, keyword search: *learning and growing together*.
- **Ohio Resource Center** – The Ohio Resource Center for Mathematics, Science and Reading (ORC) provides links to peer-reviewed instructional resources that a panel of Ohio educators has identified as exemplifying best or promising practices. Available resources also include content and professional resources as well as assessment and general education resources that will support the work of prekindergarten through grade 12 classroom teachers and higher education faculty members. The resources are correlated with Ohio's academic content standards and with applicable national content standards. <http://www.ohiorc.org>
- **Resources for Early Childhood** - The REC website seeks to support Ohio's early childhood teachers, parents, teachers of early childhood teachers, and children by providing peer-reviewed Web-based resources to assist with instructional planning, professional learning, and implementing the Ohio Early Learning Content Standards for mathematics, science, English language arts and social studies. The REC is a dynamic Web site with a rotation of new book recommendations, parent connections, and classroom resources featured on the home page. Best practice articles are published each month to support educators and enrich their teaching. Documentation of interdisciplinary projects that have been integrated into the programs of many Ohio preschool and children's centers are also featured on the Web site. <http://www.rec.ohiorc.org>

Instructional Publications

- Althouse, R. (1994). *Investigating mathematics with young children*. New York: Teachers College Press.

- Koralek, D. (2003). *Spotlight on young children and math*. Washington, DC: National Association for the Education of Young Children.
- National Council for Teachers in Mathematics. (2000). *Principles and standards in school mathematics*. Reston, VA: National Council for Teachers in Mathematics.
- Ohio Department of Education. (2004). *Guidance for early learning content standards implementation*. Columbus: Ohio Department of Education.
- Williams, R., Cunningham, D., & Lubawy, J. (2005). *Preschool math*. Belville, MD: Gryphon House.

Professional Resources

Resources listed in this section provide access to professional organizations and public institutions, to afford educators opportunities to stay informed within their fields.

Professional Organizations

- **National Association for the Education of Young Children (NAEYC)** – NAEYC exists for the purpose of “leading and consolidating the efforts of individuals and groups working to achieve healthy development and constructive education for all young children. Primary attention is devoted to assuring the provision of high quality early childhood programs for young children.” This site provides access to information on conferences and forums that provide professional development along with resources that address early mathematics and other relevant issues. <http://www.naeyc.org>
- **Ohio Department of Education**, Office of Early Learning and School Readiness. <http://www.ode.state.oh.us>, search keywords: *early learning*.
- **U. S. Department of Education**, <http://www.ed.gov>

Research Resources

Resources listed in this section provide theory, skills and strategies for building knowledge and understanding of standards and other related topics.

Research Publications

- Copley, J. V. (2000). *The young child and mathematics*. Washington, DC: National Association for the Education of Young Children.
- Kamii, C. K., & Housman, L. B. (1999). *Young children reinvent arithmetic: Implications of Piaget’s theory*. (2nd Ed.). New York: Teachers College Press.
- National Association for the Education of Young Children and the National Council for Teachers of Mathematics. (2002). *Early childhood mathematics: Promoting good beginnings*. Washington, D.C: NAEYC.

Children's Picture Books

The children's literature titles listed below are examples of useful picture books for teaching early mathematics.

- *1, 2, 3 to the Zoo* by Eric Carle
- *10 Black Dots* by Donald Crews
- *26 Letters and 99 Cents* by Tana Hoban
- *Anno's Counting Book* by Mitsumasa Anno
- *Brown Rabbit's Shape Book* by Allan Baker
- *Count* by Denise Fleming
- *Each Orange Had 8 Slices* by Paul Giganti, Jr.
- *Eating Fractions* by Bruce McMillan
- *Feast for 10* by Cathryn Falwell
- *Fish Eyes* by Lois Ehlert
- *Five Little Monkeys Jumping on the Bed* by Eileen Christelow
- *How Many Snails?* by Paul Giganti, Jr.
- *I See Patterns* by Linda Benton
- *Inch by Inch* by Leo Lionni
- *Let's Count It Out, Jesse Bear* by Nancy Carlstrom
- *Numbears* by Kathleen Hague
- *One Bear at Bedtime* by Mick Inkpen
- *One Gorilla* by Atsuko Morozumi
- *Over on the Farm* by Christopher Gunson
- *Pretty Patterns* by Rosemary Irons
- *Ten Little Rabbits* by Virginia Grossman
- *The Icky Bug Counting Book* by Jerry Pallotta
- *Too Many Monkeys* by Kelly Oechsli
- *What Comes in 2's, 3's, 4's* by Suzanne Aker

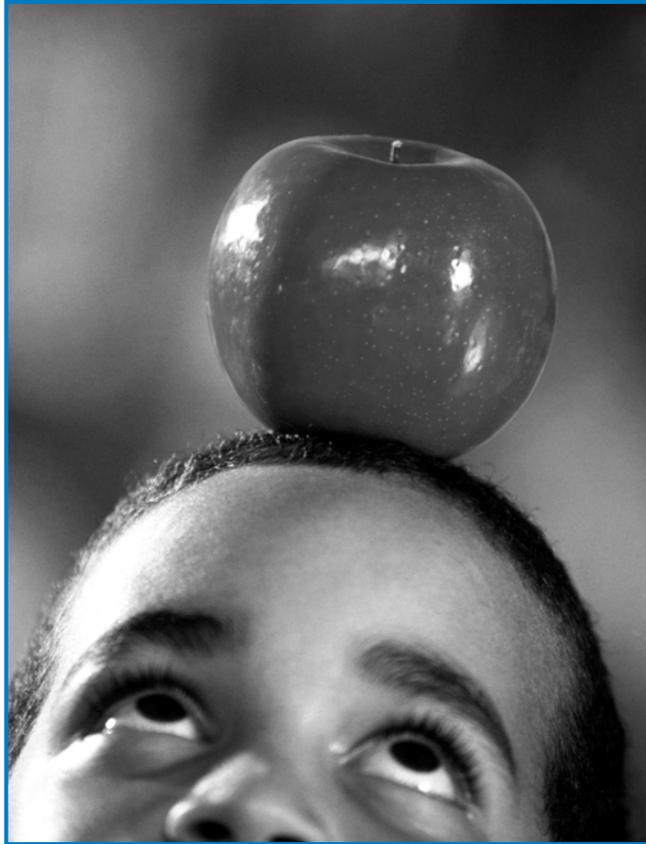
The heart or spirit must be as active as the brain if purposeful learning is to last a lifetime.

– C. Lewis, 1995



Like growing flowers, where certain specific conditions are provided to produce beautiful blossoms...adults provide the conditions that establish the growing ground for empowered children.

– Wasserman, 1990



Many things we need can wait. The child cannot. Now is the time his bones are formed, his mind developed. To him, we cannot say tomorrow, his name is today.

– Gabriela Mistral





all children are
born ready
to learn



relationships
are
influential



communication
is
critical



environments
matter



Office of Early Learning and School Readiness

25 South Front Street
Columbus, Ohio 43215-4183
(614) 466-0224; Toll-Free (877) 644-6338
www.ode.state.oh.us