Zenbat azpimultzo egon daitezke multzo batean?

Demagun $A = \{1,2\}$

k	*A*-ren azpimultzoa	Azpimultzo kopurua
0	$\emptyset$	1
1	{1} {2}	2
2	{1,2}	1
		4

$$A = \left\{1,2\right\}$$

$$P\left\{A\right\} = \left\{\left\{\emptyset\right\},\left\{\left\{1\right\},\left\{2\right\}\right\}\left\{\left\{1,2\right\}\right\}\right\}$$

$$\left|P\left\{A\right\}\right| = 4$$

A multzoaren azpimultzoen multzoa

P multzoaren kardinala

Zenbat azpimultzo egon daitezke multzo batean?

Demagun $A = \{1,2,3\}$

k	*A*-ren azpimultzoa	Azpimultzo kopurua
0	$\emptyset$	1
1	{1} {2} {3}	3
2	{1,2} {1,3} {2,3}	3
3	{1,2,3}	1
		$8 = 2^3$

$$A = \{1,2,3\}$$

$$P\{A\} = \{\{\emptyset\}, \{\{1\}, \{2\}, \{3\}\}\{\{1,2\}\{1,3\}, \{2,3\}\}\{\{1,2,3\}\}\}$$

$$|P\{A\}| = 8$$

Zenbat azpimultzo egon daitezke multzo batean?

Demagun A = {1,2,3,4}

k	*A*-ren azpimultzoa	Azpimultzo kopurua
0	Ø	1
1	{1} {2} {3} {4}	4
2	{1,2} {1,3} {1,4} {2,3} {2,4} {3,4}	6
3	{1,2,3} {1,2,4} {2,3,4} {1,3,4}	4
4	{1,2,3,4}	1
		$16 = 2^4$

$$A = \{1,2,3\}$$

$$P\{A\} = \{\{\emptyset\},\{\{1\},\{2\},\{3\},\{4\}\}\{\{1,2\}\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}\{\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,3,4\}\}\{\{1,2,3,4\}\}\}$$

$$|P\{A\}| = 16$$

Zenbat azpimultzo egon daitezke multzo batean?

Demagun $A = \{1,2,3,4\}$

k	*A*-ren azpimultzoa	Azpimultzo kopurua
0	$\emptyset$	1
1	{1} {2} {3} {4}	4
2	{1,2} {1,3} {1,4} {2,3} {2,4} {3,4}	6
3	{1,2,3} {1,2,4} {2,3,4} {1,3,4}	4
4	{1,2,3,4}	1
		$16 = 2^4$

Zenbat azpimultzo egon daitezke multzo batean?

$$2^n$$

Demostrazioa

Zenbat azpimultzo egon daitezke multzo batean?

Demagun $A = \{1,2,3,4\}$

$|P\{A\}|$ = 0 elementuko azpimultzo-kopurua + 1 elementuko azpimultzo-kopurua +...+ n elementuko azpimultzo-kopurua

$$|P\{A\}| = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}$$

$$(a+b)^n = \binom{n}{0} a^n \cdot b^0 + \binom{n}{1} a^{(n-1)} \cdot b + \binom{n}{2} a^{(n-2)} \cdot b^2 + \ldots + \binom{n}{n-1} a \cdot b^{(n-2)} + \binom{n}{n} b^n$$

a = 1 = b

$$|P(A)| = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = (1+1)^n = 2^n$$