Math 1205 Ch. 3 Problem Solving (Sec. 3.1)

Sec. 3.1 Ratios and Proportions

Ratio – comparison of two quantities with the same units
Ex.: 2 cups to 6 cups

Rate – comparison of two quantities with different units
Ex.: $40 for 3 calculators

Unit Ratios and ***Unit Rates*** – have a denominator of 1
Ex.: $2.30 per (1) gallon

1. Write as a ratio in simplest form:
   a. $12 to $18

   b. $3\frac{1}{2}$ cups to $1\frac{3}{4}$ cups

2. Write as a unit rate:
   a. $15.35 for 5 gallons of gas

   b. 186 miles in 3 hours

Math 1205 Ch. 3 Solving Problems (Sec. 3.1)

Solving Proportions

Proportion – 2 rates or ratios equal to each other

In a true proportion the cross products are _____.

Ex: $\frac{3}{4}=\frac{6}{8}$ becomes $3 \cdot 8 = 6 \cdot 4$ when cross-multiplied.

Solve:

1. $\frac{5}{8}=\frac{x}{40}$

2. $\frac{7}{8}=\frac{x-3}{16}$

3. $\frac{18}{x-4}=\frac{3}{10}$

4. $\frac{15}{x+3}=\frac{3}{x-1}$

5. Eighteen ceramic tiles are required to tile a 12 ft^2 area. At this rate, how many square feet can be tiled using 324 ceramic tiles?

6. A caterer estimates that 2 gal. of fruit punch will serve 30 people. How much additional fruit punch is needed to serve 75 people?

7. Sally is 5 ft 3 in tall and casts a 7 ft shadow at the same time that a tree casts a 59.5 ft shadow. How tall is the tree?

8. A gallon of paint covers 300 ft^2. How many gallons are needed to paint 2 coats of paint on a floor that measures 25 ft by 30 ft?

Sec. 3.2 Percents

Percent – out of 100

1. Write as a decimal:
   a. 65% b. 0.06%

2. Write as a fraction:
   a. $16\frac{2}{3}\%$ b. $242\frac{6}{7}\%$

3. Write as a percent:
   a. 0.075 b. $\frac{5}{11}$

   c. 1.23 d. $2\frac{1}{3}$

Math 1205 Ch. 3 Solving Problems (Sec. 3.2)

Translating Percent Sentences

Translate into an equation and then solve:

1. What is 1.6% of 85?

2. What number is $9\frac{1}{11}$% of 88?

3. 6 is what percent of $7\frac{1}{5}$?

4. 75% of what is 6?

5. 121.04 is 68% of what number?

Math 1205 Ch. 3 Solving Problems (Sec. 3.2)

Application Problems

When solving percent word problems, it may be useful to first fill in this template: _____% of _____ is _____

6. Approximately 21% of air is oxygen. Using this estimate, find how many liters of oxygen there are in a room containing 25,400 L of air.

_21_% of _25,400__ is ___x___
(air) *(oxygen)*

$.21 \cdot 25{,}400 = x$

x = _____ liters oxygen

7. Of the people working for a downtown bank, 88% take public transportation to work. If 484 bank employees take public transportation, how many people work at the bank?

8. A lamp was discounted 30%. If the original price was $89, what was the amount of discount and what was the price of the lamp after the discount?

9. In some restaurants a 15% tip is automatically added to the cost of the meal for large groups. If the cost of a meal for 8 people was $132, (a) what was the amount of the tip? (b) What was the total cost of the meal?

10. Terri, a restaurant server, receives a tip of $7.76. If this was 20% of the cost of the meal, what was the cost of the meal?

Math 1205 Ch. 3 Solving Problems (Sec. 3.2)

Percent Increase/Decrease Problems

_____% of *original amt.* is *amt of increase/decrease*

Solve, rounding to the nearest tenth of a percent:

11. The price of a calculator decreased from \$17.75 to \$15.50. What was the percent decrease?

12. The price of gas increased overnight from \$2.049 to \$2.249. What was the percent increase?

13. Toy sales in a department store increased from \$3500 in November to \$5800 in December. What was the percent increase?

Sec. 3.3 Problems with Two or More Unknowns

Translate and Solve (Number Relationship Problems)

Use one statement to represent the unknowns. Use the other statement to write the equation.

1. The second of two numbers is four times the first. Twice the first number is equal to thirty less than the second number. Find the numbers.

2. The larger of 2 numbers is 8 more than the smaller number. The sum of the numbers is 22. Find the numbers.

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

3. One serving of Special K Protein Plus has 100 fewer milligrams of sodium than one serving of Cheerios. If Jan eats one serving of each cereal, she would take in 320 mg of sodium. How many milligrams of sodium are in one serving of Special K Protein Plus?

Perimeter

Perimeter = all of the sides added together

1. The perimeter of a rectangle is 76 m. The length of the rectangle is 5 m more than twice the width. Find the length and the width of the rectangle.

2. The width of a rectangle is 40% of the length. The perimeter of the rectangle is 266 ft. Find the length and the width of the rectangle.

3. The perimeter of a rectangle is 42 m. The width is 3 m less than the length. Find the width.

4. In an isosceles triangle, two sides are equal. The third side is 5 m less than one of the equal sides. The perimeter is 40 m. Find the length of each side.

5. The perimeter of a triangle is 59 ft. One side of the triangle is 2 ft longer than the second side. The third side is 3 ft longer than the second side. Find the measure of each side.

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

Angles

Complementary angles – 2 angles whose measures total 90°
Supplementary angles – 2 angles whose measures total 180°

1. Two angles are complementary. Three times the first angle is four more than twice the second angle. Find the measures of the angles.

2. Two angles are supplementary. The smaller angle is 32° less than the larger angle. Find the measures of the angles.

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

Consecutive Integers

Consecutive integers – positive or negative whole numbers that occur in sequence, such as: 3, 4, 5 or -7, -6, -5

Represent them as follows:

1^{st} : ____

2^{nd} : _____

3^{rd} : _____

1. The sum of three consecutive integers is -510. Find the integers.

   1^{st} : _____

   2^{nd}: _____

   3^{rd}: _____

2. Find 2 consecutive integers such that four times the first is fourteen more than three times the second.

   1^{st}: ____

   2^{nd}: _____

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

Consecutive Even or Odd Integers

Ex: 1, 3, 5 or 4, 6, 8

Represent them as follows:

1st : _____

2nd : _____

3rd : _____

Why are these represented in the same way?

3. Four times the smallest of 3 consecutive even integers is four more than twice the largest. Find the integers.

1st:____

2nd: _____

3rd: _____

4. Three times the smallest of 3 consecutive odd integers is three more than twice the largest. Find the integers.

1st: ____

2nd: _____

3rd: _____

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

Number Value Problems

Formula: Price of One x Number = Total Amount of $

These problems are more easily solved with the use of a table (like a mini-spreadsheet).

1. Blue sweatshirts sell for $23.95 and white ones sell for $18.95. If a total of 54 sweatshirts sell for a total of $1173.30, how many of each color were sold?

	Price of 1	Number	Total Amt of $
Blue			
White			
Totals			

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

2. The Licking River 4-H Club is having its annual fundraising dinner. Adults pay $15 each and children pay $10. If the number of adult tickets sold is three times the number of children's tickets sold, and the total income for the dinner was $2200, how many of each kind of ticket did the club sell?

	Price of 1	Number	Total Amt of $
Adults			
Children			
Totals			

Math 1205 Ch. 3 Solving Problems (Sec. 3.3)

3. Peter has some $5 bills and some $10 bills in his pocket. If he has a total of 20 bills, which total $115 in value, how many of each kind of bill does he have?

	Price of 1	Number	Total Amt of $
$5 bills			
$10 bills			
Totals			

4. There were 708 people at an organ recital. Orchestra seats cost $8.00 each and balcony seats cost $5.00 each. The total receipts were $4431. Find the number of orchestra seats and the number of balcony seats sold.

	Price of 1	Number	Total Amt of $
Orch.			
Balc.			
Totals			

Math 1205 Ch. 3 Solving Problems (Sec. 3.4)

Sec. 3.4 Rates (Distance Problems)

Formula: **R**ate · **T**ime = **D**istance (where rate = speed)

1. Two small planes start from the same point and fly in opposite directions. The first plane is 30 mph slower than the second plane. In 5 hours the planes are 1950 mi apart. Find the rate of each plane.

	rate	time	distance
1st			
2nd			
totals	//////////		

Hints for distance problems:

1. There is never a "total rate" (bottom left).
2. If a total distance is needed to solve the problem, it will be given.
3. If the problem asks for a distance, find it as an extra step at the end of the problem.

Math 1205 Ch. 3 Solving Problems (Sec. 3.4)

2. Two cyclists start at the same time from opposite ends of a course which is 70 mi long. One cyclist is riding at 16 mph and the second is riding at 12 mph. How long after they begin will they meet?

	rate	time	distance
1st			
2nd			
totals	////////		

3. An executive flew in a helicopter to the airport to board a plane. The helicopter's flying speed was 120 mph and the airplane's flying speed was 650 mph. The entire trip was 2335 mi and took 4 hours. How far did the executive fly in the helicopter?

	rate	time	distance
helicopter			
plane			
totals	////////		

Math 1205 Ch. 3 Solving Problems (Sec. 3.4)

4. A motorboat leaves a harbor going 10 mph toward a small island. Two hours later a speed boat leaves the same harbor and travels at 18 mph toward the same island. In how many hours after the speed boat leaves will the speed boat be alongside the motorboat?

	rate	time	distance
motorboat			
speedboat			
totals	////////		

5. A motorcycle and a bicycle start at 8 am, from the same point, traveling in the same direction. The motorcycle's speed is 3 times the speed of the bicycle. In 2 hours the motorcycle is 80 miles ahead of the bicycle. Find the rate of each.

	rate	time	distance
motorcycle			
bicycle			
totals	////////		

Math 1205 Ch. 3 Solving Problems (Sec. 3.5)

Sec. 3.5 Investment and Mixture Problems

Simple Interest Problems

Investment Formula: **P**rincipal · **R**ate = **I**nterest
(rate = percent at which money is invested)

1. A total of $8000 is deposited into two simple interest accounts. One account pays 5%, while the other account pays 6%. How much should be invested in each account so that the total interest earned is $450?

	Principal	Rate	Interest
1st			
2nd			
Totals			

2. Jill invests 40% of her money at 4% annual simple interest and the rest at 6%. At the end of one year the total interest earned was $1560. What was the total amount she invested?

	Principal	Rate	Interest
1st			
2nd			
Totals			

Math 1205 Ch. 3 Solving Problems (Sec. 3.5)

3. Jim invested three-fourths of his money into a simple interest account paying 7% and the rest into a CD paying 5%. If his total interest income for the year was $338, how much did he invest in each account?

	Principal	Rate	Interest
1st			
2nd			
Totals			

4. An accountant deposited some money into a 5% simple interest account. Another deposit, $4000 more than the first, was placed in a $2\frac{1}{2}\%$ account. The total interest earned on both investments for 1 year was $550. How much money was deposited into the 5% account?

	Principal	Rate	Interest
1st			
2nd			
Totals			

Math 1205 Ch. 3 Solving Problems (Sec. 3.5)

Percent Mixture Problems

Formula: **A**mount x **R**ate = **Q**uantity
(where **R**ate = % concentration)

1. A chemist has some 12% hydrogen peroxide solution and some 9% hydrogen peroxide solution. How many milliliters of each should be mixed to make a 510 milliliter solution which is 11% hydrogen peroxide?

	Amount	Rate	Quantity of Pure Stuff
12% sol'n			
9% sol'n			
11% sol'n			

Hints for Percent Mixture Problems:
All 9 cells in the spreadsheet get filled in.
Pure water is _____% salt, alcohol, whatever
Pure salt is _____% salt; Pure alcohol is _____% alcohol

Math 1205 Ch. 3 Solving Problems (Sec. 3.5)

2. How many grams of pure acid must be added to 240 g of a 15% acid solution to make a solution that is 40% acid?

	Amount	Rate	Quantity of Pure Stuff
pure acid			
15% sol'n			
40% sol'n			

3. How many ounces of water must be added to 150 oz of a 30% salt solution to make a salt solution that is 20% salt?

	Amount	Rate	Quantity of Pure Stuff
water			
30% sol'n			
20% sol'n			

Math 1205 Ch. 3 Solving Problems (Sec. 3.5)

Value Mixture Problems

Formula: **A**mount x Unit **C**ost = **V**alue

Unit Cost = price per lb or price per kg etc

1. A grocer combined candy corn costing \$2.60/lb with peanuts costing \$3.20/lb. How many pounds of each were used to make a 36 lb mixture to sell for \$3.00/lb?

	Amount	Unit Cost	Total Amt of \$
candy corn			
peanuts			
mixture			

2. How many bushels of corn worth \$2.00/bu must be mixed with 1400 bu of soybeans worth \$6.00/bu to make a mixture worth \$5.00/bu?

	Amount	Unit Cost	Total Amt of \$
corn			
soybeans			
mixture			

Math 1205 Ch. 3 Solving Problems (Sec. 3.5)

3. A 120-lb mixture consists of 2 grades of tea, one costing \$1.20/lb and the other \$1.60/lb. How many pounds of each kind are in the mixture if it sells for \$1.42/lb?

	Amount	Unit Cost	Total Amt of \$
Tea 1			
Tea 2			
mixture			

4. A delicatessen owner mixed coffee which cost \$4.50/lb with coffee which cost \$3.00/lb. How many pounds of each were used to make a 10 lb blend costing \$3.60/lb?

	Amount	Unit Cost	Total Amt of \$
Coffee 1			
Coffee 2			
mixture			